From 'p-TKE' to 'e-TKE': suppressing restrictive conditions, validating the extension of the ALARO-0 approach and exploring links with other methods

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eTKE = extension of pTKE

pTKE scheme [6] is based on pseudo prognostic TKE equation, which enables space consistent variation (advection and auto-diffusion of TKE) around static solution $\widetilde{E}$, obtained by Louis scheme [7]:

$$\frac{dE}{dt} = -\frac{\partial}{\partial z}\left(-K_E\frac{\partial E}{\partial z}\right) + \frac{1}{\tau_\epsilon}\left(\widetilde{E} - E\right), \quad (1)$$

$$\widetilde{E} = \left(\frac{\sqrt{K_m K_N}}{\nu l_m}\right)^2, \; \tau_\epsilon = \frac{l_m^2}{\nu^2\sqrt{K_m K_N}}, \; K_E = \frac{\sqrt{K_m K_N}}{\nu^2}, \quad (2)$$

where K_m is vertical exchange coefficient for momentum, K_N is vertical exchange coefficient at neutrality, l_m is Prandtl-type mixing length for momentum and $\nu = C_K.C_\epsilon$ is a constant.

eTKE scheme (emulation TKE) is an extension of pTKE scheme. eTKE keeps time-step organisation, staggering and solver of pTKE, but uses TKE closure scheme to compute static solution $\widetilde{E}$ and variables τ_ϵ and K_E.

eKTE is implemented through new expressions for stability functions $F_m(Ri)$, $F_h(Ri)$ ($K_m = K_N F_m(Ri)$, $K_h = C_3 K_N F_h(Ri)$, C_3 is inverse turbulent Prandtl number at neutrality, Ri is gradient Richardson number). This approach enables to keep shallow convection parametrisation and antifibrilation scheme (with adequate modification) like in pTKE.

Stability functions $F_m(Ri)$, $F_h(Ri)$

The derivation of stability function $F_m(Ri)$, $F_h(Ri)$ is based on equivalence of non-discretised pseudo prognostic TKE equation and full TKE equation , which can be written as:

$$\widetilde{E} = \frac{E}{\epsilon}(I + II), \quad (3)$$

where I and II are shear production and buoyancy production terms and ϵ is dissipation.

The link between TKE closure scheme and similarity laws(K-closure) is provided by RMC01-type derivation [8] with more general relation for momentum flux (stability function $\chi_3(Ri)$).

Expression for stability function ϕ_m (non-dimensional gradient of wind shear) is adapted from [4]: $\phi_m = \chi_3(Ri)^{-\frac{1}{2}} f(Ri)^{-\frac{1}{4}}$, where $f(Ri) = \chi_3(Ri)(1 - Ri_f)$, Ri_f is flux Richardson number.

Resulting stability functions are:

$$F_m(Ri) = \chi_3\sqrt{\chi_3(Ri) - RiC_3\phi_3(Ri)}, \quad F_h(Ri) = F_m(Ri)\frac{\phi_3(Ri)}{\chi_3(Ri)}. \quad (4)$$

We also get new relations for τ_ϵ and K_E :

$$\tau_\epsilon = \frac{l_m^2}{\nu^2\sqrt{K_m K_N}}\frac{\chi_3(Ri)^{\frac{3}{2}}}{f(Ri)^{\frac{3}{4}}}, \; K_E = \frac{\sqrt{K_m K_N}}{\nu^2}\frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri)^{\frac{3}{2}}}. \quad (5)$$

Stability functions $\chi_3(Ri)$, $\phi_3(Ri)$

We tested 3 sets of stability functions $\chi_3(Ri)$, $\phi_3(Ri)$:

1. CBR scheme [5] (**CBR**): $\chi_3(Ri) = 1$, $\phi_3(Ri) = \frac{f(Ri)}{f(Ri)+C_4 Ri}$ (C_4 is a constant),
2. modified CCH scheme [4] (**CCH_mod**) (modification to avoid existence of critical gradient Richardson number Ri_{cr}),
3. fited QNSE scheme [9] (**QNSE_fit**).

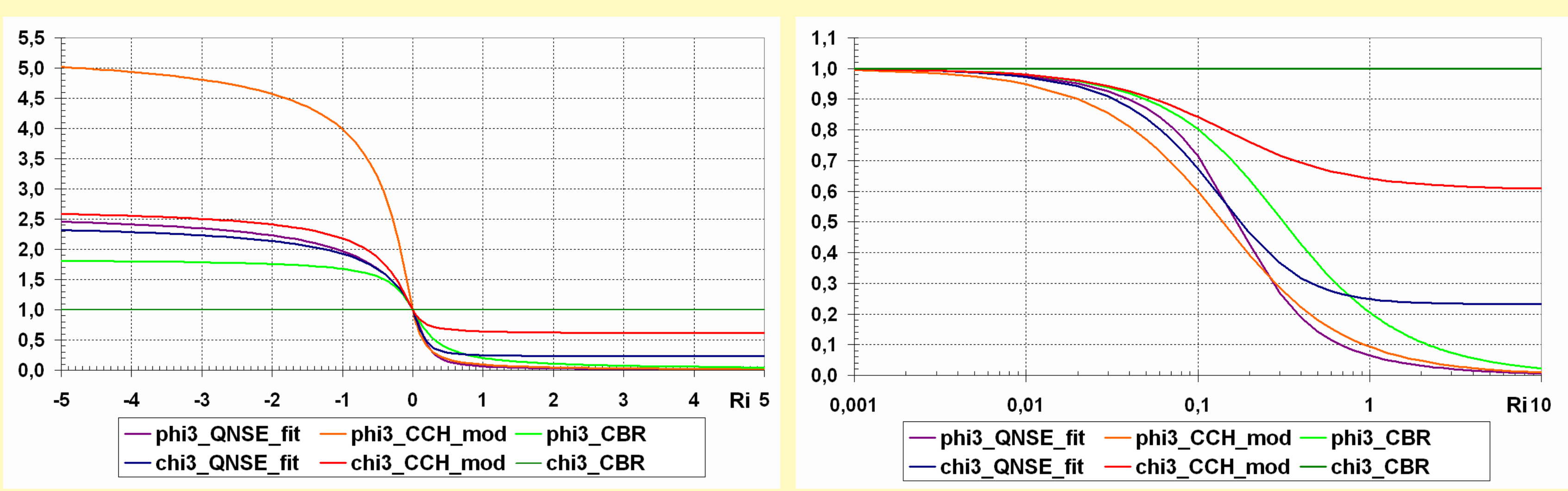

Fig. 1, Stability functions $\chi_3(Ri)$ and $\phi_3(Ri)$.

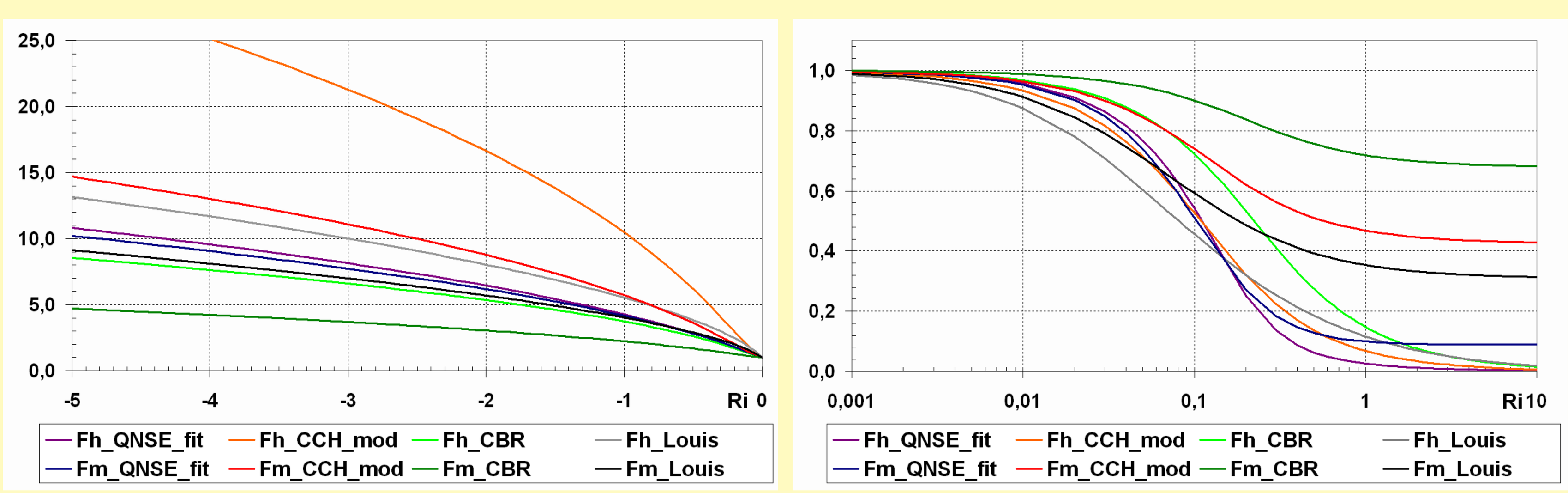

Fig. 2, Stability functions $F_m(Ri)$ and $F_h(Ri)$.

Mixing lengths

The computation of mixing length l_m is independent on TKE equation. eTKE can use TKE-based mixing length L to compute Prandtl-type mixing length l_m^{TKE} via conversion relation:

$$l_m^{TKE} = \frac{C_3}{C_4 C_\theta}\nu^3 L, \quad (6)$$

where C_4 and C_θ are constants.

We tested 2 TKE-based mixing lengths:

$$L_{BL}(E') = \left(\frac{L_{up}^{-\frac{4}{5}} + L_{down}^{-\frac{4}{5}}}{2}\right)^{-\frac{5}{4}} \quad [3] \text{ and} \quad (7)$$

$$L_N(E') = \sqrt{\frac{2E'}{N^2}}, \quad (8)$$

$$\text{where} \quad E' = \alpha_{TKE}E.$$

L_{up} (L_{down}) represents the distance that a parcel originating from the given level, and having initial kinetic energy equal to the mean TKE of the layer, can travel upward(downward) before being stopped by buoyancy effects, N is Brunt–Väisälä frequency and α_{TKE} is a tunable degree of freedom.

We have 5 optional possibilities how to compute mixing length l_m:
3 pure mixing lengths: l_m^{BL}, l_m^N and l_m^{Louis} and 2 hybrid mixing lengths (after [2]):

$$l_m^{BL,Louis} = \left(\frac{1}{l_m^{Louis}} + \frac{1}{l_m^{BL}}\right)^{-1}, \quad l_m^{N,Louis} = \left(\frac{1}{l_m^{Louis}} + \frac{1}{l_m^{N}}\right)^{-1}.$$

SCM experiment

eTKE was tested in SCM variant (Single Column Model) of ALADIN/ARPEGE model version CY34 with 60s time step and 64 vertical levels. The test confirmed equivalence between eTKE and scheme with full prognostic TKE equation.

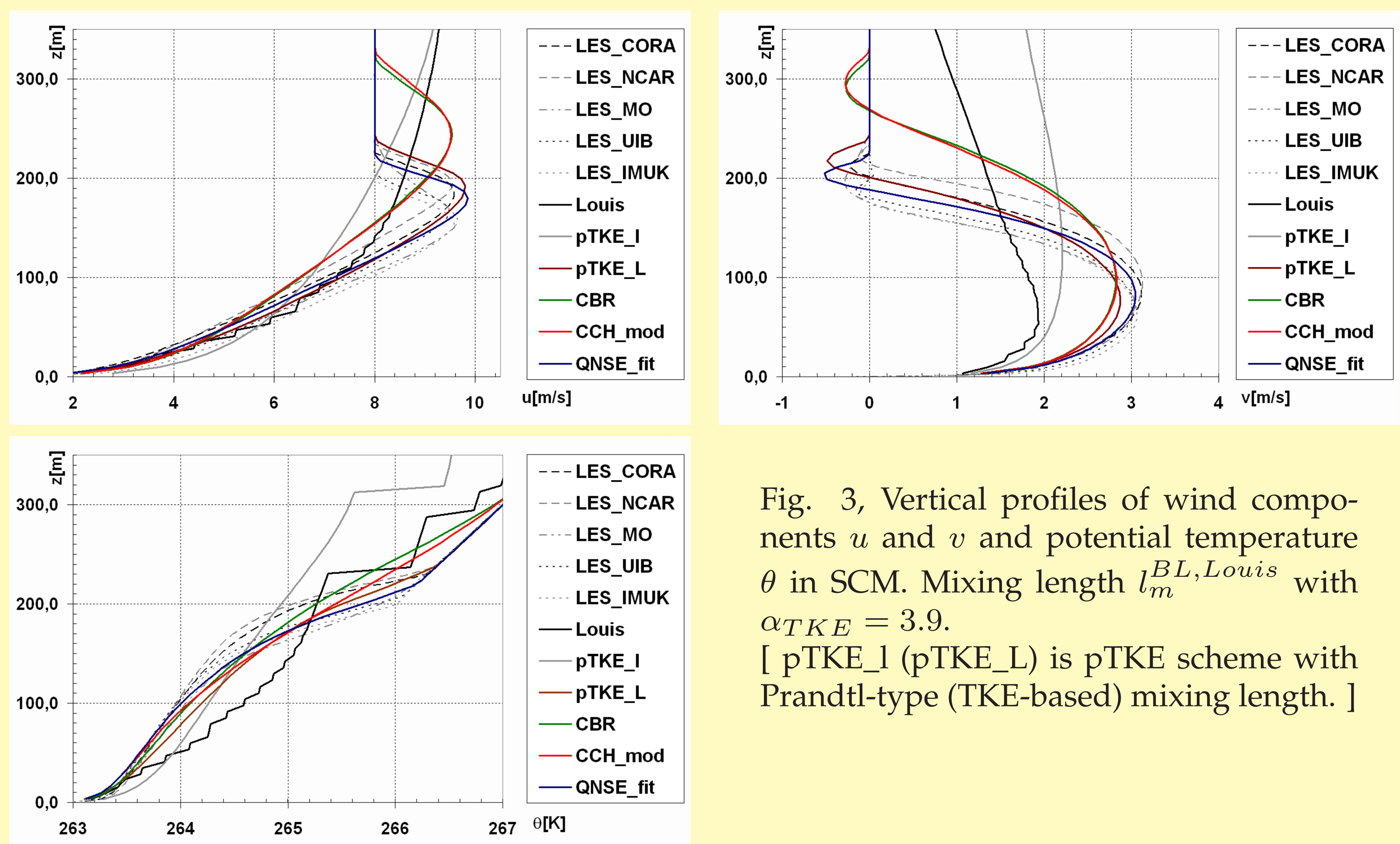

Fig. 3, Vertical profiles of wind components u and v and potential temperature θ in SCM. Mixing length $l_m^{BL,Louis}$ with $\alpha_{TKE} = 3.9$.
[pTKE_l (pTKE_L) is pTKE scheme with Prandtl-type (TKE-based) mixing length.]

The comparison with LES models (GABLS project [1]) shows that, eTKE predicts realistic vertical profiles of wind and potential temperature (Fig. 3). The best prediction of the boundary layer depth and elevated inversion is achieved with emulation of QNSE scheme.

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