Report on

“Xu-Randall Probability Density Function Distribution”

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1. Introduction

The goal of the present stay is to find a simple way of introducing the convective cloudiness independently of the exact shape of any probability density function (PDF). During the first of the two planned stays, some sensitivity tests were made to the modified Xu-Randall scheme as well as research with the bibliography listed below, namely on Betchold *et al.* (1995). In the second period of two weeks the method described below was implemented and tested using the operation version of the model in operations at CHMI (cycle 32t1alr02).

In section 2 the general method for computing total cloudiness and condensate is described, section 3 presents the details and results regarding the fitting of the data model. In section 4 some details about the implementation of the method are made as well as cloud and condensate data analysis, followed by some final remarks in section 5.

2. Ideas on cloudiness

The general method for computing the total cloudiness for radiation is as follows:

a) Define variable Q_1 as

$$Q_1 = \frac{HU - 1}{1 - HU_c} \quad (1)$$

where HU is the saturation ratio given by

$$HU = \frac{q_v + q_l + q_i}{q_w} \quad (2)$$

where q_v is the vapour content, q_l is the cloud water content, q_i is the cloud ice content and q_w is the saturation value. HU_c is the critical humidity value.

b) In Bechtold *et al.*(1995), a parameterization of the fractional cloudiness is given as a function of the variable Q_1, that is $N=f(Q_1)$.

c) Once the stratiform cloudiness (N_{st}) has been computed by the Xu-Randall modified scheme, the issue is how to add the convective cloudiness. Let N_{cv} be the convective cloudiness from the previous time-step. **Total Cloudiness** (N_t) can then be defined as

$$N_t = N_{cv} + (1 - N_{cv})N_{st} \quad (3)$$

Equation 3 complies immediately with the following two basic requirements:

1) if there is no convective cloudiness, the total one is equal to the stratiform;

2) if there is no stratiform cloudiness, the total one is equal to the convective.

Computation of the total cloudiness is therefore trivial, once the function f is known. Even though it was not applied in this study, another possible formulation for the total cloudiness is given by equation (3a).

$$N_t = (1 - N_{st})N_{cv} + (1 - (1 - N_{st})N_{cv})N_{st} \quad (3a)$$

This equation also complies with conditions 1 and 2 and provides slightly smaller values of cloudiness when both stratiform and convective clouds occur. As they cannot be strictly separated, this formulation has the advantage of scaling the convective cloudiness from the value of the stratiform one computed in subroutine *acnebcond*.

The computation of the total condensate is a little more complex and requires the following additional steps.

d) The computation of the **Q_1 Total** (Q_1^t) assumes that q_v, q_w and HU_c are constant and is computed by equation (4).

$$Q_1^t = f^{-1}(N_t) \quad (4)$$

e) Compute the **Total Saturation Ratio** (HU_t) by the following equation, by inverting equation (1)

$$HU_t = 1 + Q_1^t(1 - HU_c) \quad (5)$$

f) The **Total Condensate** (q_c^t) can be computed by inverting equation (2),

$$q_c^t = q_l^t + q_i^t = HU_t q_w - q_v \quad (6)$$

g) Finally, the partitioning between the liquid and ice content can be made quite easily through the FONICE function.

3. Fitting model data

The model was run for three hours (thirty time-steps) so that one can observe the relationship between the stratiform cloudiness (computed in *acnebcond*) and variable Q_1.

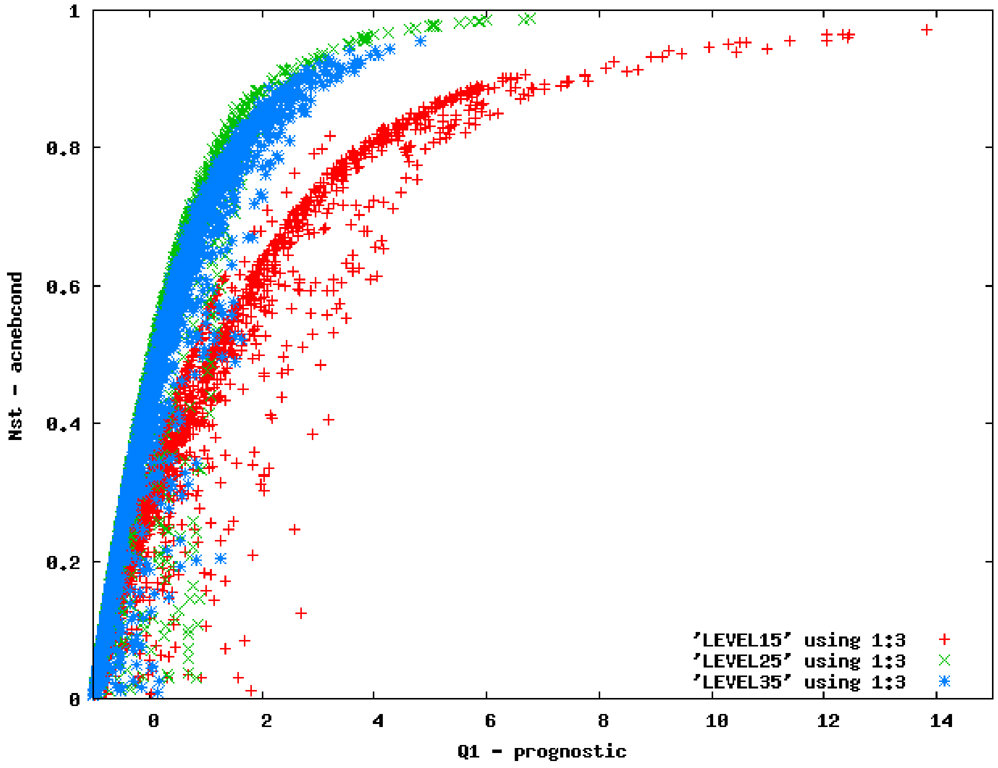


Figure 1 – Cloudiness at three model levels (15, 25, 30) as a function of Q_1.

Figure 1 shows that the relationship between stratiform cloudiness and variable Q_1 is level dependent, as the curve for the highest model level (15) is clearly different from the ones representing the lower and medium atmosphere (which have similarities).

To simulate the relationship between stratiform cloudiness and variable Q_1 several functions were used. The chosen function was the one shown is table 1. Even though the fit is level dependent, for the sake of simplicity, only two fits were used in this study, as shown in table 1.

Table 1 – Fits used in this study.

Level	**Fit:** $y = a + b\tanh(c(x+1)^d)$
Upper atmosphere (<=18)	$y = f(x) = 0.96\tanh(0.2542(x+1)^{1.05})$
Medium/lower atmosphere (>18)	$y = f(x) = 0.99\tanh(0.4742(x+1)^{0.90})$

The inverting function is then given by $x = -1 + \left(\frac{1}{2c}\ln\left(\frac{b+y-a}{b-y+a}\right)\right)^{1/d}$.

Figures 2, 3 and 4 show the relationship between stratiform cloudiness and Q_1, as well as the fit used for each of the levels.

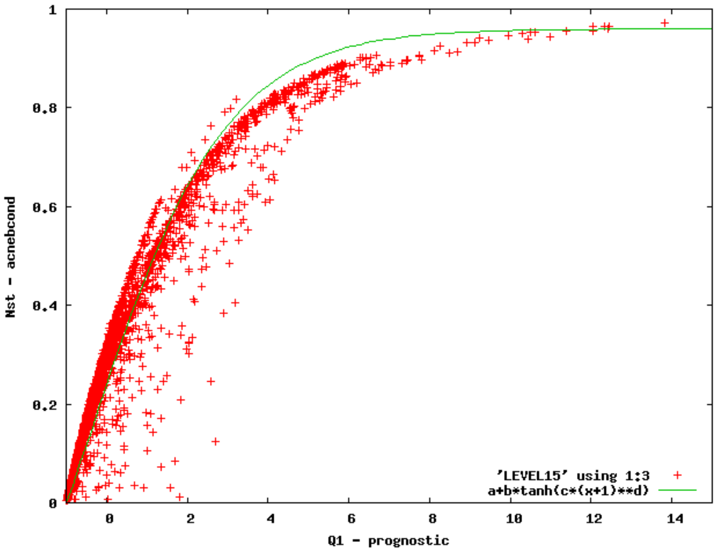


Figure 2 – Cloudiness at level 15 as a function of Q_1 and the fit used for the upper-levels.

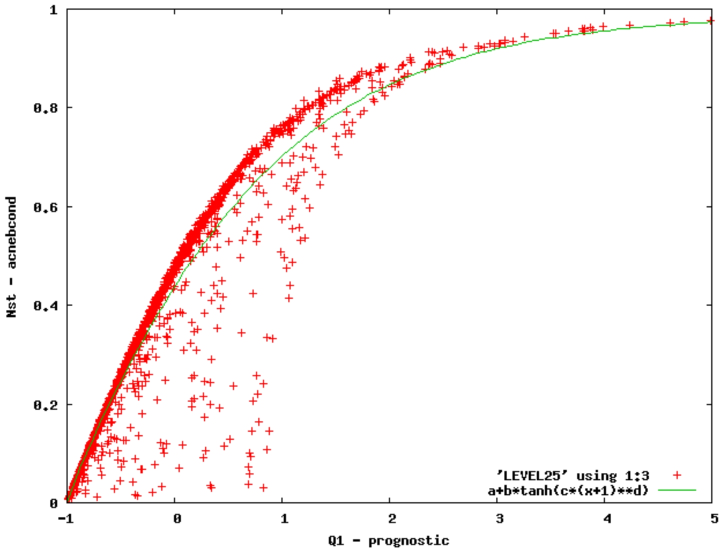


Figure 3 – Cloudiness at level 25 as a function of Q_1 and the fit used in medium/lower levels.

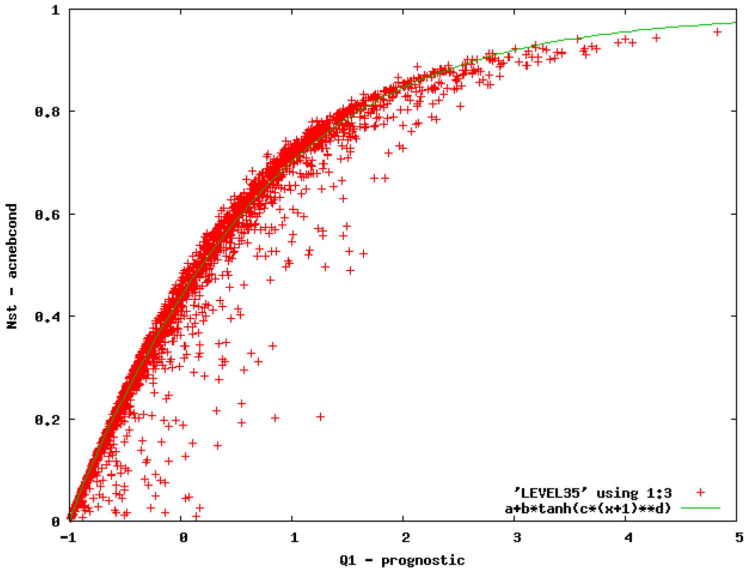


Figure 4 – Cloudiness at level 35 as a function of Q_1 and the fit used in medium/lower levels.

Analysis of the fits shows that they don't converge exactly to full cloudiness (that is, value 1). Having this in mind and to avoid any problems when inverting the fit, the cloudiness (C) is always taken as the value given by equation 7

$$C = \max(0.95, N_k) \quad (7)$$

where k stands for either stratiform or total cloudiness.

Figure 5 displays the dispersion plot between the variables Q_1 computed from:

a) the prognostic values, by $Q_1^{obs} = f^{-1}(q_l, q_i, q_v)$ (8)

b) the stratiform cloudiness, by $Q_1^{st} = f^{-1}(N_{st})$ (9).

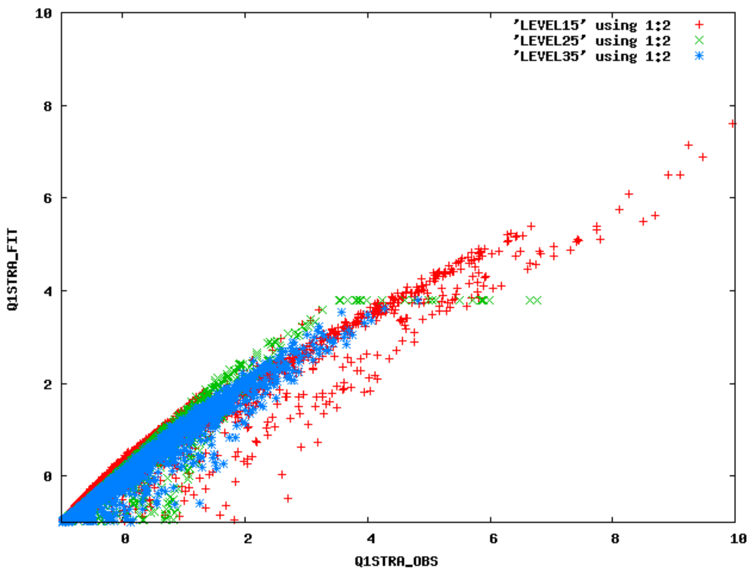


Figure 5 – Dispersion plot between Q_1 computed from the prognostic variables (*q1stra_obs*) and the stratiform cloudiness (*q1stra_fit*).

Even though there is a fairly good agreement between prognostic Q_1^{obs} and the value computed from the fit using the stratiform cloudiness (Q_1^{st}), there are outliers. A possible cause for this feature was the introduction of N_{cv} in the Newton Loop that computes the stratiform cloudiness in routine acnebcond.

To assess this hypothesis, define Δ as the difference of the values of Q_1 as given by equation (10)

$$\Delta = Q_1^{obs} - Q_1^{st} \quad (10)$$

Figure 6 shows the relationship between Δ and the convective cloudiness (variable ZUNEBH) from the previous time-step.

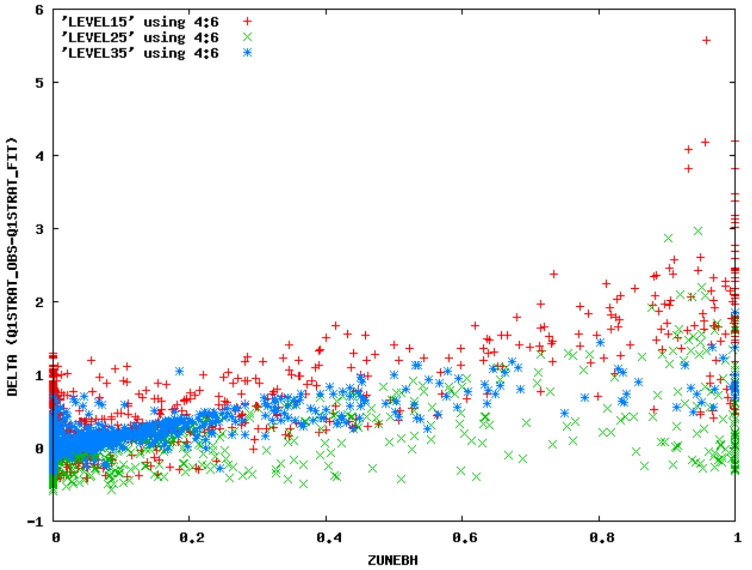


Figure 6 – Difference of Q_1 as a function of convective cloudiness (ZUNEBH).

Note that the introduction of the convective cloudiness in the Newton Loop makes Δ become negative, because the value of Q_1^{st} becomes bigger than if only the prognostic (hence, "only" stratiform) values of water species are considered. Unfortunately no straightforward conclusion can be taken from figure 6 and therefore it seems difficult to change the initial definition of Q_1, so that this variable has clear input information from the convective cloudiness.

4. Results

The model was run for July, 24th 2008, 00 UTC, because in this day and on the following days Central Europe was affected by convective systems. To make the data analysis as simple as possible, the total sample of grid-points was divided in three groups: only stratiform cloudiness exists; only convective cloudiness exists and both exist. Unless otherwise mentioned, all the results shown below are from levels 15 to 30 (all inclusive) and for a three hour forecast.

The total cloudiness was computed by equation (3). The computation of the total condensate, given by the general formula (4), is a little more complex and requires some caution. Figure 7 shows the procedure needed to compute the final Q_1 total cloudiness.

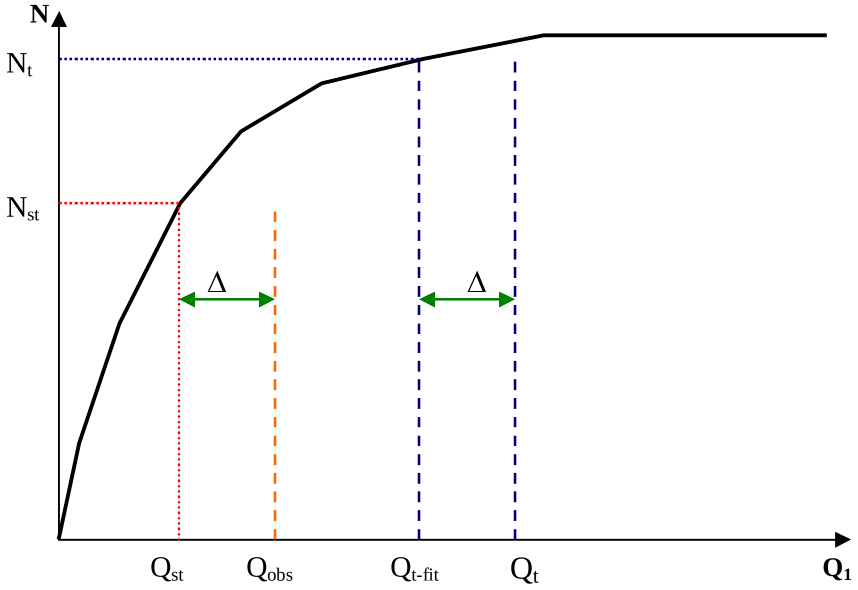


Figure 7 – Schematic view of the procedure required to compute Q_1 total.

As discussed earlier, there are non-negligible differences between the Q_1^{obs} (equation 8) and the Q_1^{st} computed from the stratiform cloudiness (equation 9), as shown by figure 6 (delta). To take this into account and after analysis of the first results, the computation of the total Q_1 (Q_1^t) is given by expression (11):

$$Q_1^t = \Delta + Q_1^{obs} + (Q_1^{t-fit} - Q_1^{st}) * ZUNEBH \quad (11)$$

where ZUNEBH, which is the convective cloudiness, is taken as a scaling factor and Q_1^{t-fit} is the variable computed from the total cloudiness (equation 3).

Finally, the computation of the total condensate has shown to be a little more complex. In fact the total condensate computed from equation 6 showed some problems in the case when the convective cloud cover is very high or when the grid-point vapour content is much lower when compared to the saturation.

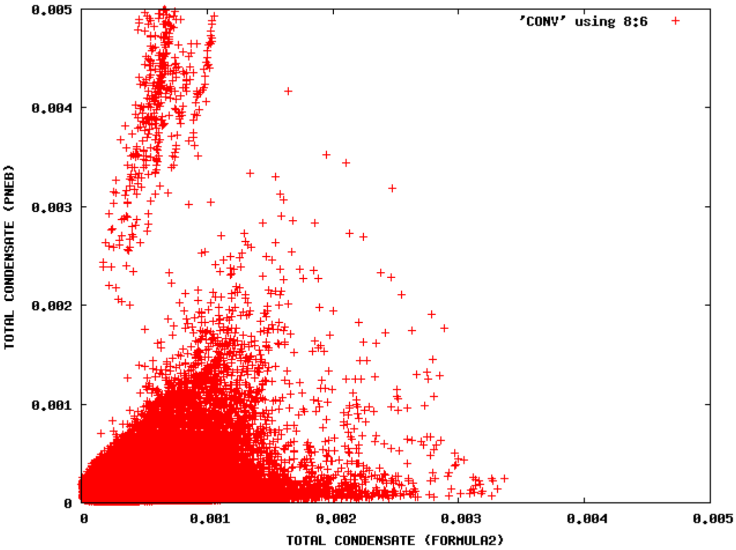


Figure 8 – Dispersion plot of total condensate between ACNEBN

and method under analysis, with the unexpected values in the upper-left.

To avoid this, a possible re-arrangement of the exact computation for the total condensate could be given by equation 12.

$$q_c^t = HU^t q_w - (ZNEBS * q_v + k(1 - ZNEBS) * q_w) \quad (12)$$

where k is a tunning constant (taken for example as 0,95). When stratiform cloudiness is very high, this expression would provide the same results as equation 6; on the other hand, when there are very few or no stratiform clouds, instead of using q_v, one should use q_w to ensure that the subtracting factor in equation 12 is not much lower than q_w. This expression may be tested later on.

The results obtained in this stay are shown in three items, depending on the type of existing clouds. Also the total amount of cloudiness and condensate are compared against the results of routine acnebcond/prognostic variables as well as the ones computed from the present diagnostic scheme (*acnebn*). All the results are taken from data between levels 15 and 30.

A) Only Stratiform clouds exist

Figures 9 to 12 show the relationship between total cloudiness and condensate, against the original values (from *acnebcond* and prognostic water content) as well as the ouput from routine *acnebn*. The sample of points considered in this plots is required to have N_{st}>0,005 and *ZUNEBH*<0,01.

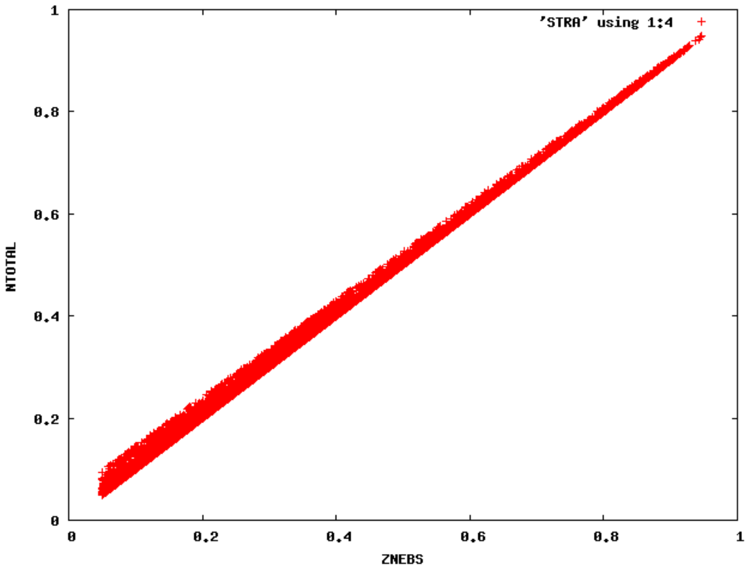


Figure 9 – Total cloudiness (ntotal) *vs* the one computed in routine *acnebcond* (ZNEBS).

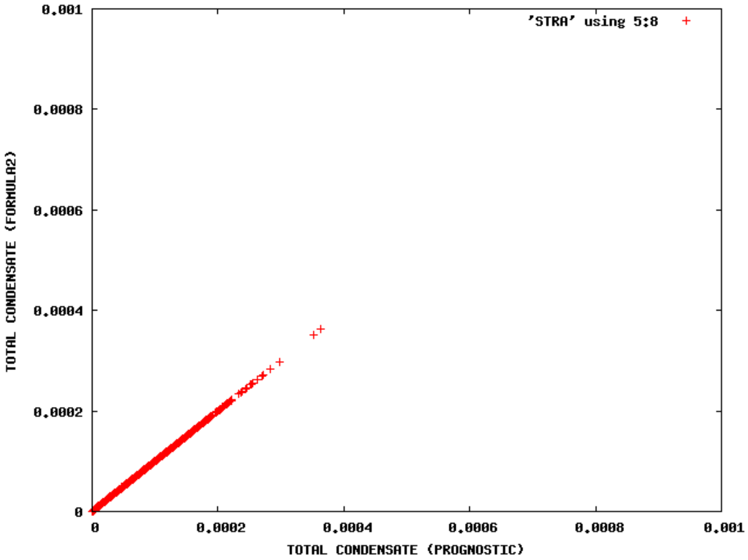


Figure 10 – Total Condensate vs the prognostic one.

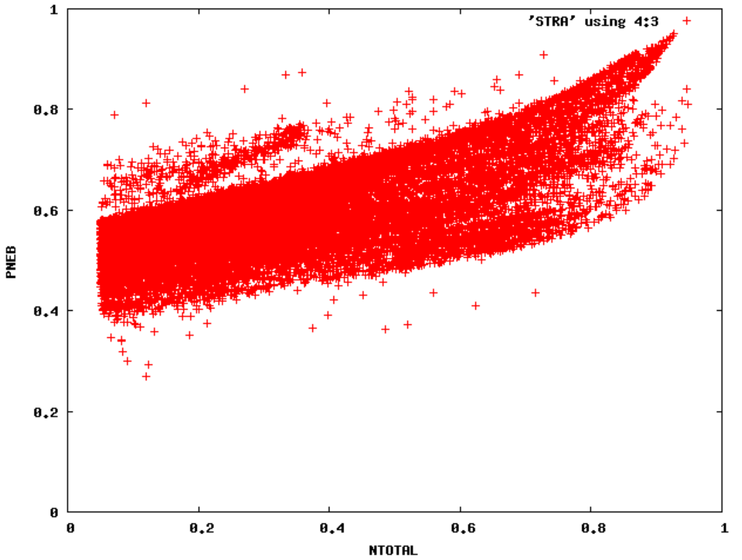


Figure 11 – Total cloudiness (ntotal) *vs* the one computed in routine *acnebn* (PNEB).

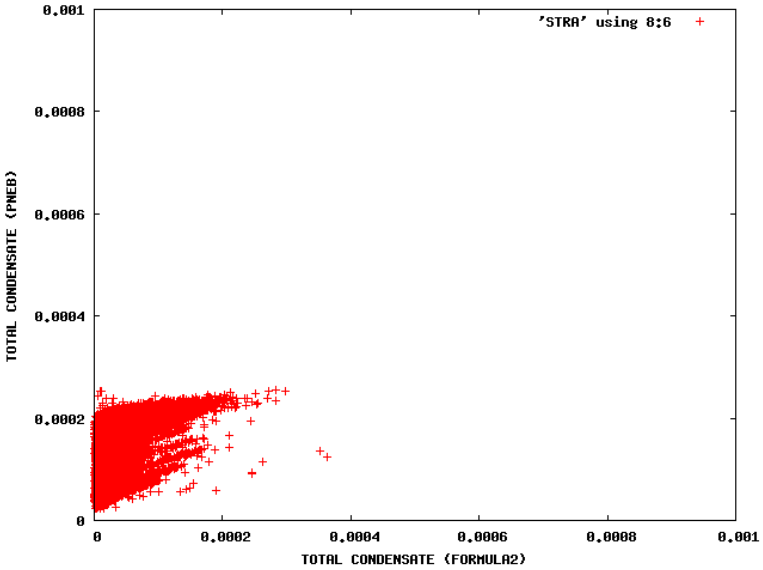


Figure 12 – Total condensate (*formula2*) *vs* the one computed in routine *acnebn*.

The analysis of figures 9 to 12 show that the method for computing the total cloudiness from the Xu-Randall pdf distribution is working correctly, because the original values are maintained. When the comparison is made against output from routine *acnebn* then it seems clear that this one is over-estimating both variables.

B) Only Convective clouds exist

Figures 13 to 15 show the relationship between total cloudiness and ZUNEBH, as well as the outputs obtained from routine *acnebn*. The sample of points considered in this plots is required to have N_{st}<0,01 and *ZUNEBH*>0,01.

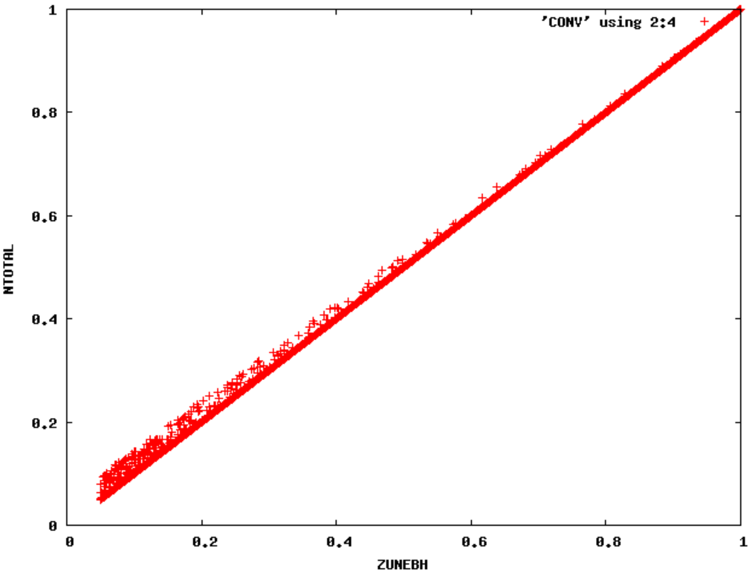


Figure 13 – Total cloudiness (ntotal) *vs* the convective one (ZUNEBH).

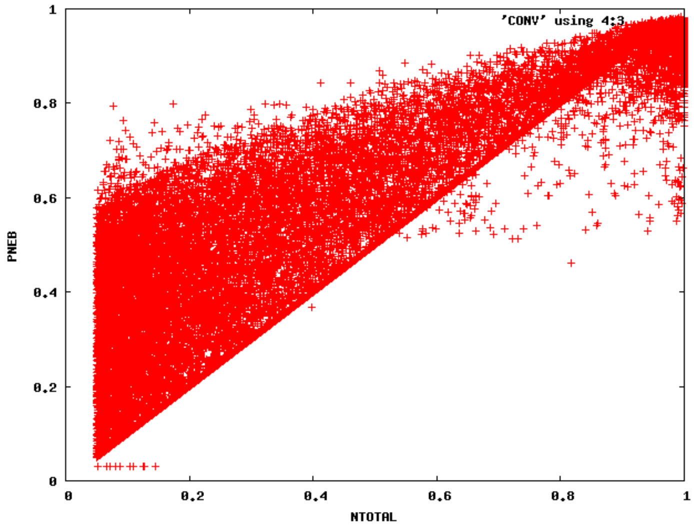


Figure 14 – Total cloudiness (ntotal) *vs* the one computed in routine *acnebn* (PNEB).

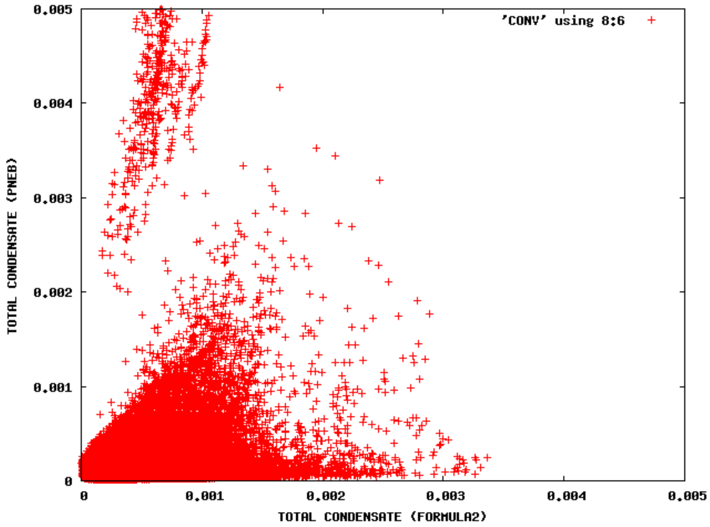


Figure 15 – Total condensate *vs* the one computed in routine acnebn.

The results are the ones expected, except for the behaviour in figure 14 when the cloud cover approaches one. In figure 15 one notices in the upper-left the very low total values of condensate (*formula2*) when compared to the ones diagnosed in routine *acnebn*.

To try to understand the existence of these points, figures 16 splits the data used to make figure 15 as a function of the level. Figure 16a restricts the data to levels between 15 and 20, 16b to levels 21 to 28 and 16c to levels 29 to 35. As one can see, the pattern is clearly visible in the upper levels of the atmosphere, becoming less important at lower levels.

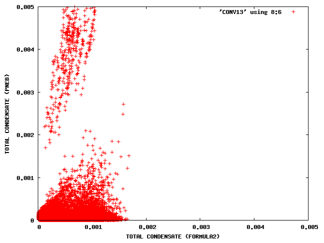


Figure 16a – levels 15 to 20

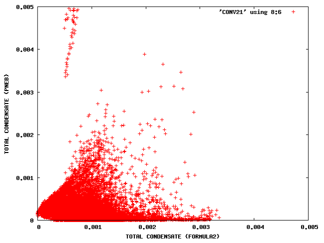


Figure 16b – levels 21 to 28

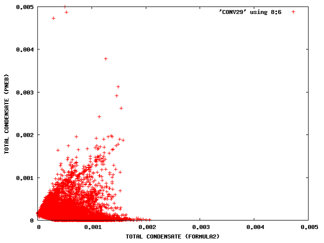


Figure 16c – levels 29 to 35

In light of these results and even though equation 12 may be helpful, one may still question if the problem is not due to the usage of an inadequate fit at high levels at the atmosphere.

C) Both Stratiform and Convective clouds exist

Figures 17 to 19 show the relationship between total cloudiness and condensate against the values obtained from routine *acnebn*. The sample of points considered in this plots is required to have N_{st}>0,01 and *ZUNEBH*>0,01. The results are basically the expected as they are a combination of conditions A) and B). Clearly seen is the fact that PNEB is producing much higher values of both condensate and clouds that the ones computed with the fit.

Additionally, figure 20 shows the type of cloud cover as diagnosed in routine *acnpart*. When the cloud cover is computed via the fit, the general features are present, but the field is clearly too spotty and apparently there is a lack of intermediate to low amounts of cloud cover.

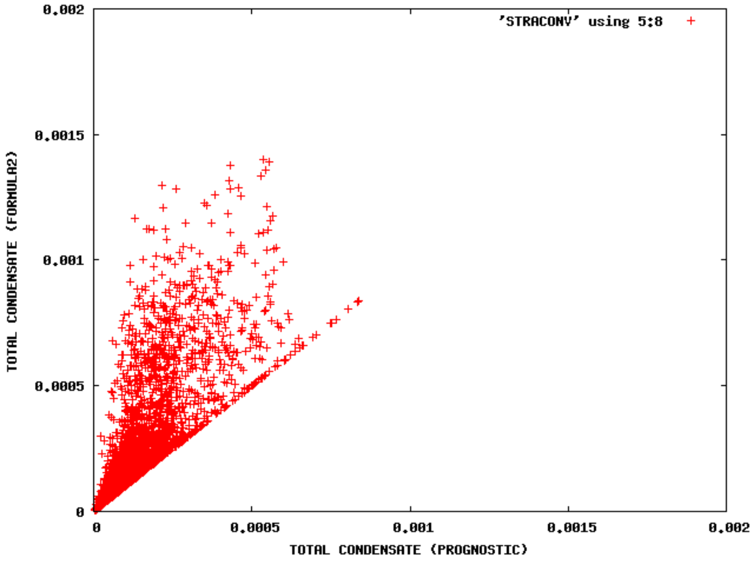


Figure 17 – Total condensate *vs* the one computed from the prognostic water species.

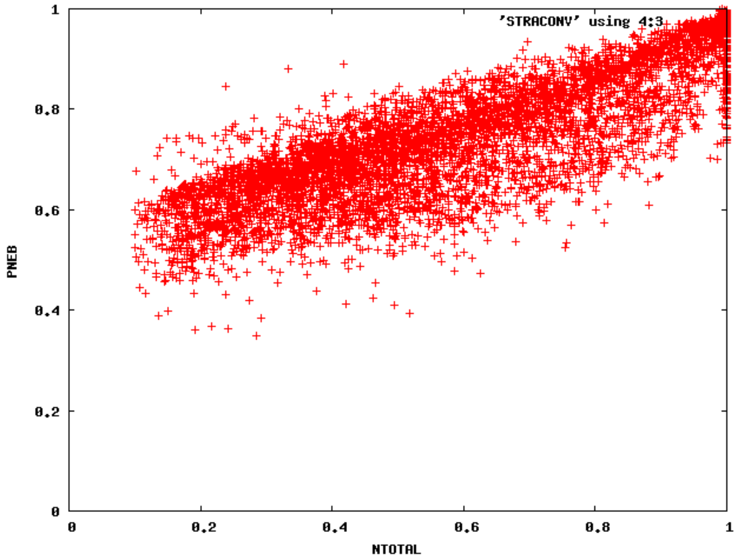


Figure 18 – Total cloudiness (ntotal) *vs* the one computed in routine acnebn (PNEB).

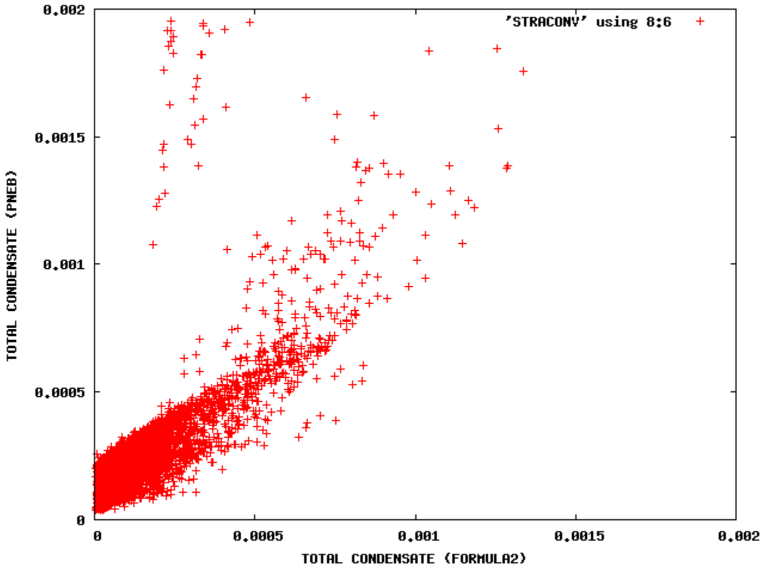
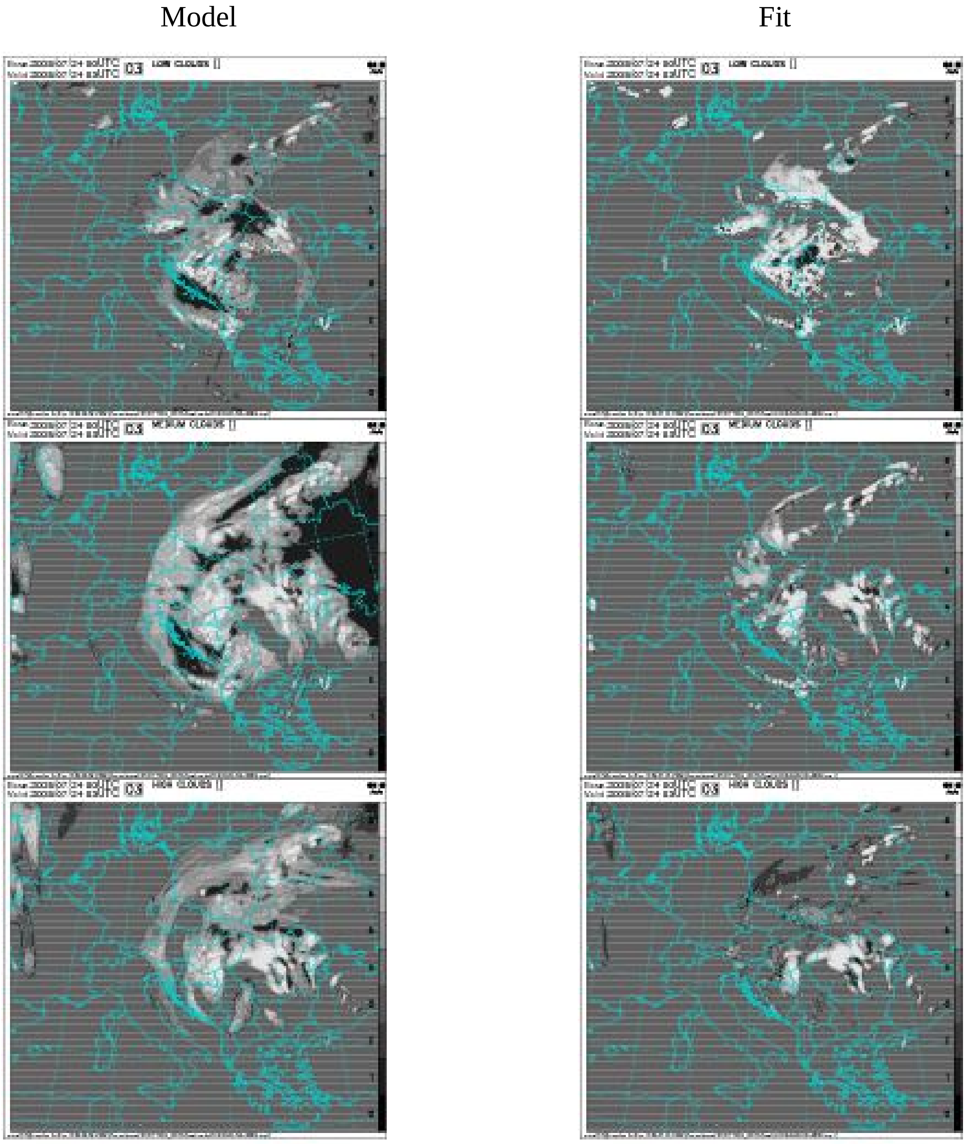


Figure 19 – Total condensate *vs* the one computed in routine *acnebn*.


Model

Fit

Figure 20 – Total cloudiness from the regular model (left) and by the fit (right) as diagnosed in routine *acnpart*.

5. Final Remarks

From these results, the following remarks can be made:

a) as a general method, the pdf distribution of the Xu-Randall modified-scheme for stratiform cloudiness seems to provide very reasonable results;

b) as it was seen, the fit is dependent on the model level. For simplicity, only two fits were considered: one for the upper and another for the medium and lower atmosphere. Therefore a more sophisticated scheme to vary the fit almost continuously could be tried in the future;

c) the introduction of the convective cloud cover is therefore feasible and, very importantly, independent of the chosen pdf distribution;

d) some modifications to the formulas used to compute the total condensate adjustments may be tested – indeed this is far more complex than the actual formulation of total cloudiness;

e) the results for the total cloudiness and condensate differ considerably from the ones obtained from the fully diagnostic procedure and hence show that some changes either in *acnebcond* or in physical parameterizations may be necessary.

Bibliography

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