B-matrix: general overview and theoretical aspects

LACE DAWD & DAsKIT Working Days

Pierre BROUSSEAU
DESR/CNRM/GMAP
Météo-France

plan

1) Introduction

2) B modeling in IFS/arpege/aladin/arome

3) B estimation

4) B variations : Flow dependency / hourly cycle

Questions

#	Question	Specific	SECTION
1	When computing a climatological B-matrix by an ensemble technique, how many members and how long should be the period to consider and why ?	Pierre	3
2	Ideally, each B-matrix should be computed for each geographical domain and model geometry. However, is it possible to short-cut B-matrix computation: i) by cutting it from a geographically larger B-matrix; ii) from a lower resolution B-matrix; iii) from a higher resolution B-matrix; from a different vertical resolution B-matrix ?	Pierre	3
3	Are there available tools to perform the operations mentioned in 2 ?	Pierre	3
4	Which are the different aspects to take into account, when computing B-matrix by an ensemble technique, if the model is coupled by either IFS/ECMWF or ARPEGE ?	Antonin	
5	Is it possible to cut a LAM B-matrix from a global B-matrix ? for instance, to cut AROME B-matrix from IFS/ECMWF ?	Pierre	2
6	In case of 5, could B-matrix be used as a first B-matrix in operations ?	Pierre	2
7	How important it is to use the same forecast length in the sampling of differences for B and in the actual DA cycling (e.g. forecasts of 3h length for 3-hourly cycle or 1h for hourly cycle)	Pierre	4
8	How much impact is expected from daily recalculation of B (in 3D-Var) if a real-time ensemble is available? Is it worth the effort or is it better to plan to set up the EnVar in this case?	Pierre	4
9	What is the impact of different B-matrices over different types of observations.	Antonin	
10	How much is it worth to invest on B-matrix before implementing a local operational DA system	Antonin	
11	What is the role and NLEVBAL0 and NLEVBAL1 parameters for limiting the vertical extend of balances in B?	Pierre	4

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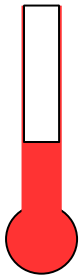
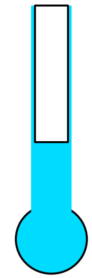
Linear estimation theory

information : 2 measurements T_1 et T_2

$$T_1 = T_t + \varepsilon_1,\quad E(\varepsilon_1) = 0,\quad E(\varepsilon_1^2) = \sigma_1^2$$



$$E(\varepsilon_1 \varepsilon_2) = 0$$



$$T_2 = T_t + \varepsilon_2,\quad E(\varepsilon_2) = 0,\quad E(\varepsilon_2^2) = \sigma_2^2$$

Best
Linear
Unbiased
Estimate

$$T_a = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} T_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} T_2$$

$$= T_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} (T_1 - T_2)$$

Minimise the objective function

$$J(T) = \frac{(T - T_1)^2}{\sigma_1^2} + \frac{(T - T_2)^2}{\sigma_2^2}$$

Data assimilation

- **Linear Estimation Theory : the Best Linear Unbiased Estimate**

$$T_a = T_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(T_1 - T_2)$$

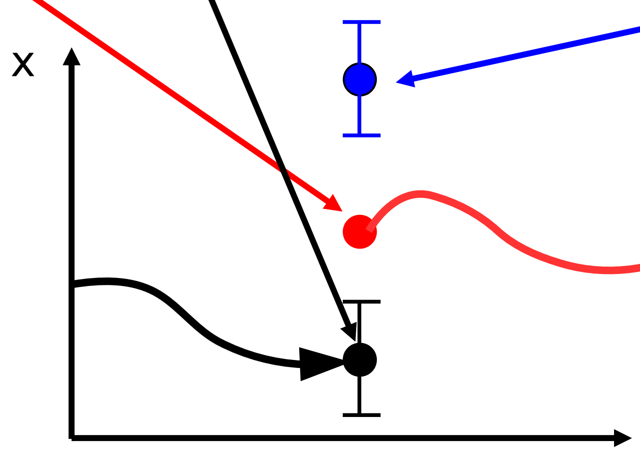
$$\mathbf{x}^a = \mathbf{x}^b + \underbrace{\mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}}(\mathbf{y}^o - H(\mathbf{x}^b))$$

Analysis

Background

$\mathbf{K}$: Kalman gain

Observations



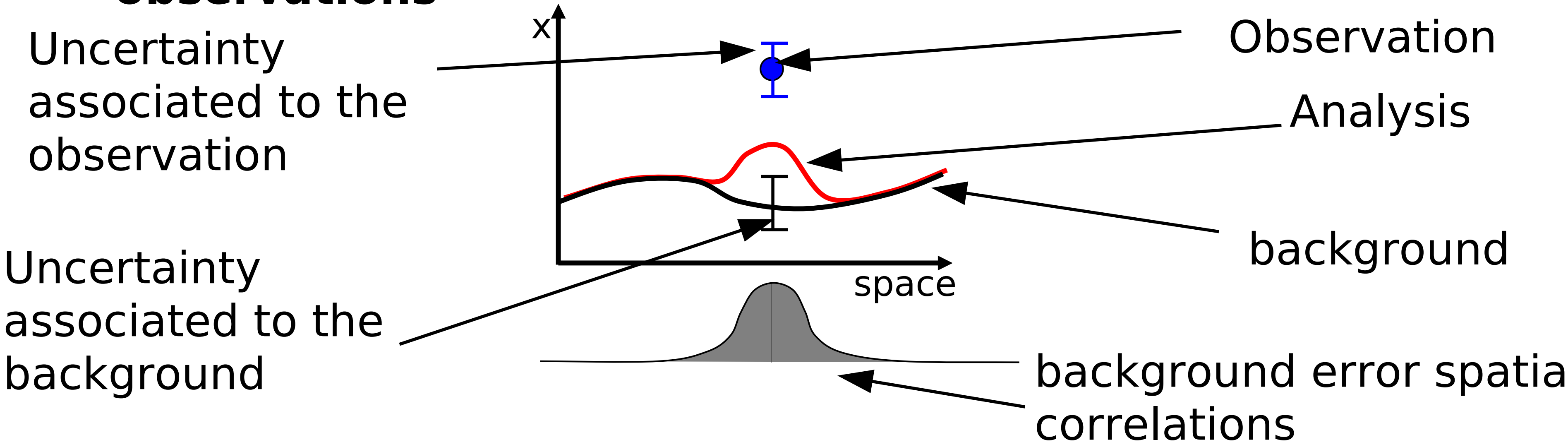
$\mathbf{B}$ Matrix of background error-covariances

$\mathbf{R}$ Matrix of observations error-covariances

H Observation operator

The B matrix role

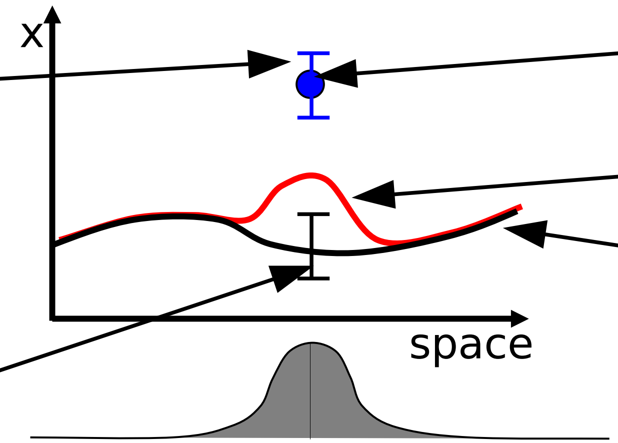
- **B propagates and filters the information provided by the observations**



The B matrix role

- **B propagates and filters the information provided by the observations**

Uncertainty associated to the observation



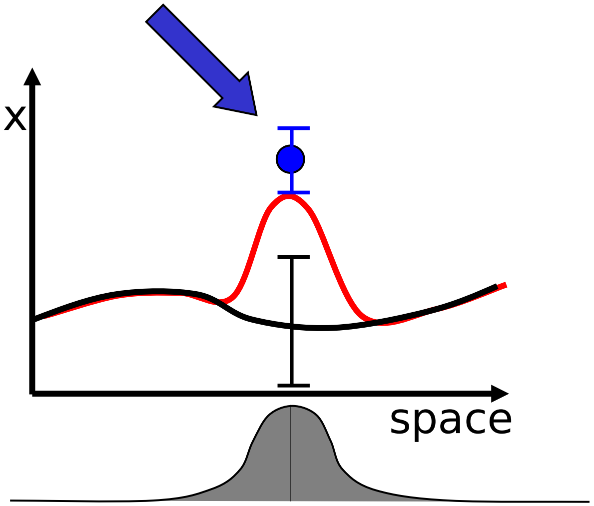
Observation

Analysis

background

background error spatial correlations

Stonger Uncertainty associated to the background



The B matrix role

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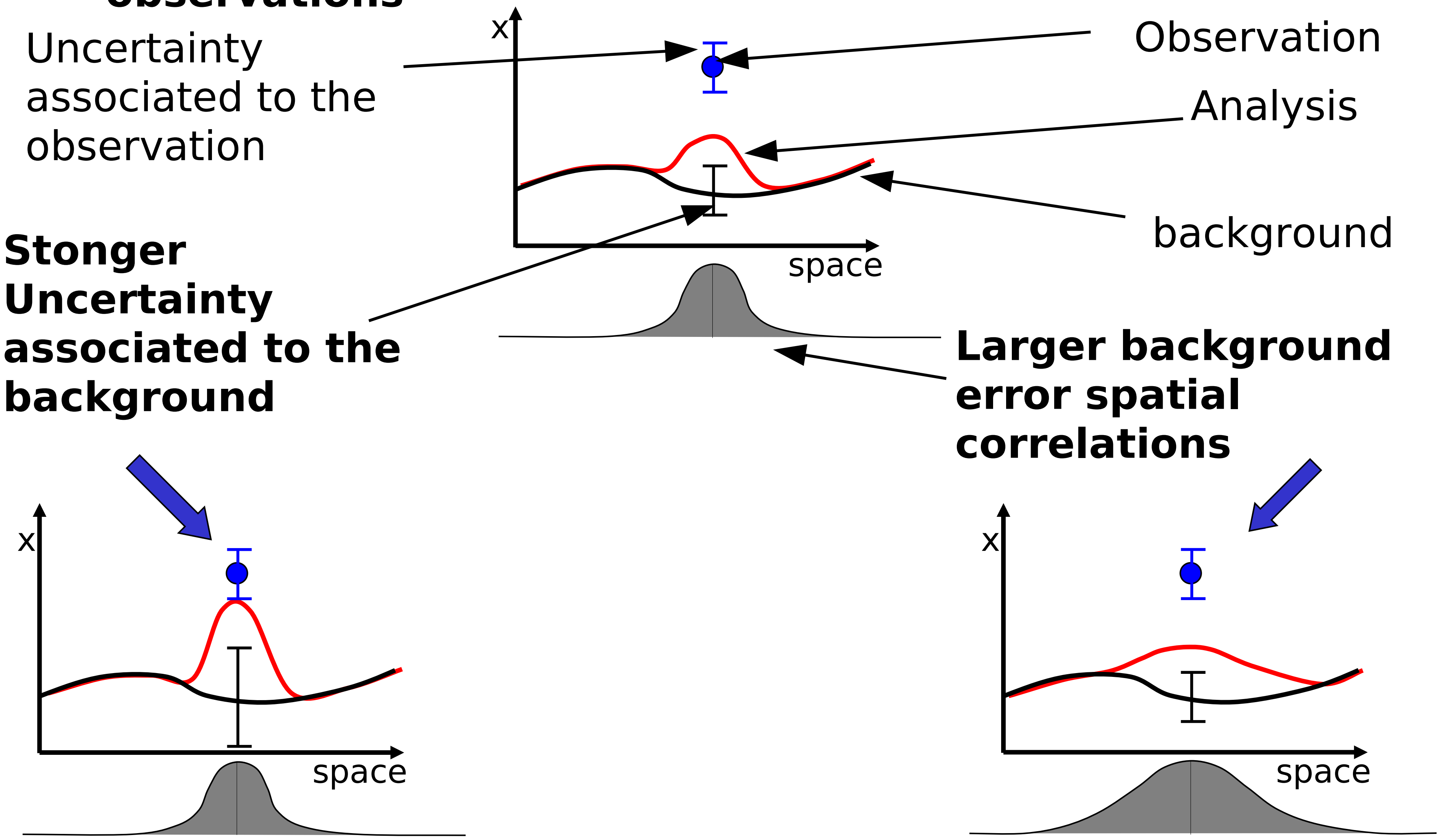

The B matrix role

→ B determines :
- the intensity of the bakground modification at the observation location (σ^b)
- how this modification is propagated on the horizontal and the vertical (correlations)
- how these modifications are propagated on the others variable of the control variable (cross-correlations)

→ B should depend on :
- the model and its resolution (Stefanescu et al 2006 : arpege Vs aladin, Brousseau et al 2011 : arome Vs aladin)
- the geographical area (mid-latitude Vs tropical, sea Vs mountain, …)
- the meteorological situation (Berre et al. 2007, Brousseau et al. 2012)
- the density of observation network (Belo-pereira and Berre 2006)

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B modeling

- **1st difficulty** : due to the large size of the system in NWP models, B can not be explicitly written and stored : B is modelized by different operators : Parrish et al. 1997, Derber and Bouttier 1999 for global model, *Berre* 2000 for LAM under some asumptions (homogeneity, isotropy, stationnarity,...

Vorticity	ζ	$=$	ζ
Divergence	η	$=$	$\mathcal{M}\mathcal{H}\zeta + \eta_u$
Mass field	(T, P_s)	$=$	$\mathcal{N}\mathcal{H}\zeta + \mathcal{P}\eta_u + (T, P_s)_u$
Specific Humic	q	$=$	$\mathcal{Q}\mathcal{H}\zeta + \mathcal{R}\eta_u + \mathcal{S}(T, P_s)_u + q_u$

Berre 2000 : multivariate formulation for q in LAM

$\mathcal{H}$ is an horizontal balance operator

$\mathcal{M}, \mathcal{N}, \mathcal{P}, \mathcal{Q}, \mathcal{R}$ and $\mathcal{S}$ are vertical balance operators relating vertical profiles of predictors and of predictands.

B modeling

$$\mathrm{B} = \overline{\mathrm{B}} = \mathbf{K}\mathbf{B}_u\mathbf{K}^T$$

$$\mathbf{B}_u = \begin{pmatrix} \mathbf{C}_\zeta & 0 & 0 & 0 \\ 0 & \mathbf{C}_{\eta_u} & 0 & 0 \\ 0 & 0 & \mathbf{C}_{(T,P_s)_u} & 0 \\ 0 & 0 & 0 & \mathbf{C}_{q_u} \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} I & 0 & 0 & 0 \\ \mathcal{M}\mathcal{H} & I & 0 & 0 \\ \mathcal{N}\mathcal{H} & \mathcal{P} & I & 0 \\ \mathcal{Q}\mathcal{H} & \mathcal{R} & \mathcal{S} & I \end{pmatrix}$$

$\mathbf{C}$ corresponds to vertical error covariances matrices for each predictor, with one matrix per total spectral wavenumber, I is the identity matrix and subscript T denotes transposition.

Question 5 : Is it possible to cut a LAM B-matrix from a global B-matrix ? for instance, to cut AROME B-matrix from IFS/ECMWF ?

$$\mathrm{B} = \overline{\mathrm{B}} = \mathbf{K B_u K^T}$$

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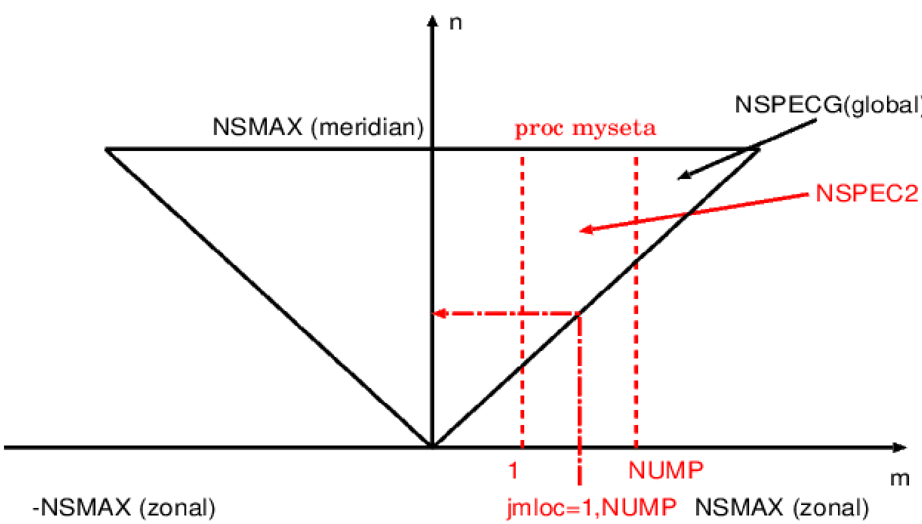
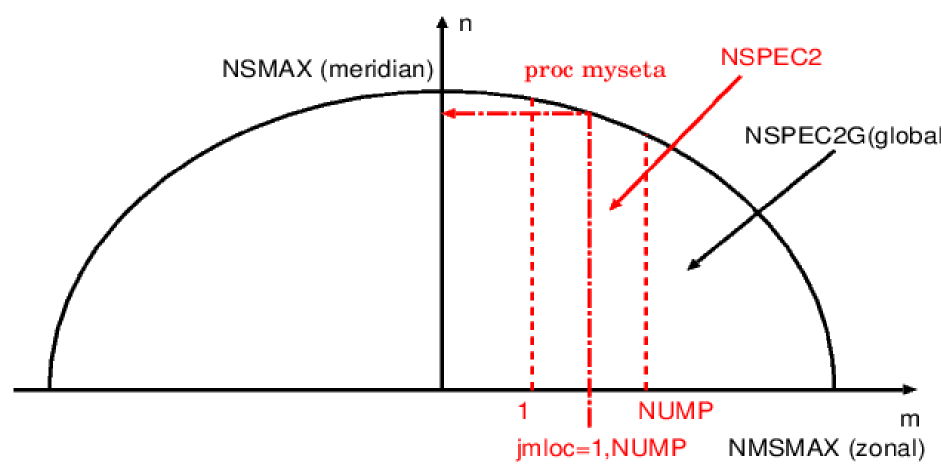
Need for change from triangular (global) to elliptic (LAM) truncation





Question 5 : Is it possible to cut a LAM B-matrix from a global B-matrix ? for instance, to cut AROME B-matrix from IFS/ECMWF ?

$$\mathrm{B} = \overline{\mathrm{B}} = \mathbf{K}\mathbf{B}_{\mathbf{u}}\mathbf{K}^{\mathrm{T}}$$

$$\mathbf{B}_u = \begin{pmatrix} \mathbf{C}_\zeta & 0 & 0 & 0 \\ 0 & \mathbf{C}_{\eta_u} & 0 & 0 \\ 0 & 0 & \mathbf{C}_{(T,P_s)_u} & 0 \\ 0 & 0 & 0 & \mathbf{C}_{q_u} \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} I & 0 & 0 & 0 \\ \mathcal{M}\mathcal{H} & I & 0 & 0 \\ \mathcal{N}\mathcal{H} & \mathcal{P} & I & 0 \\ \mathcal{Q}\mathcal{H} & \mathcal{R} & \mathcal{S} & I \end{pmatrix}$$

Univariate specific humidity in global while $\mathcal{R}$ reaches 20 % and $\mathcal{S}$ 30 % of the total varaince for q in LAM

$$\mathbf{K} = \begin{pmatrix} I & 0 & 0 & 0 \\ \mathcal{M}\mathcal{H} & I & 0 & 0 \\ \mathcal{N}\mathcal{H} & \mathcal{P} & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$

I don't know if a tool can be written to convert triangular truncation to an elliptic one, I'm sure that Q, R and S for LAM can't be derived from 0 in global …

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B estimation

- **2nd difficulty : the true state of the atmosphere is unknown : true background errors are also unknown :** proxi of background error are obtained from forecast differences :
  - differences between forecasts started from successive analyses and valid at the same time : NMC method (parrish et al. 1997
  - differences from an ensemble assimilation (EDA) Fisher (2003), more realistic than the NMC method, thanks to
  a better representation of data density effects in particular (Berre et al. 2006)

Question 1 : When computing a climatological B-matrix by an ensemble technique, how many members and how long should be the period to consider and why ?

- Mathematical constraint : positive defined B matrix needs for a number of forecast differences higher than the number of vertical levels :
  - B of the day : few assimilation times x numerous members

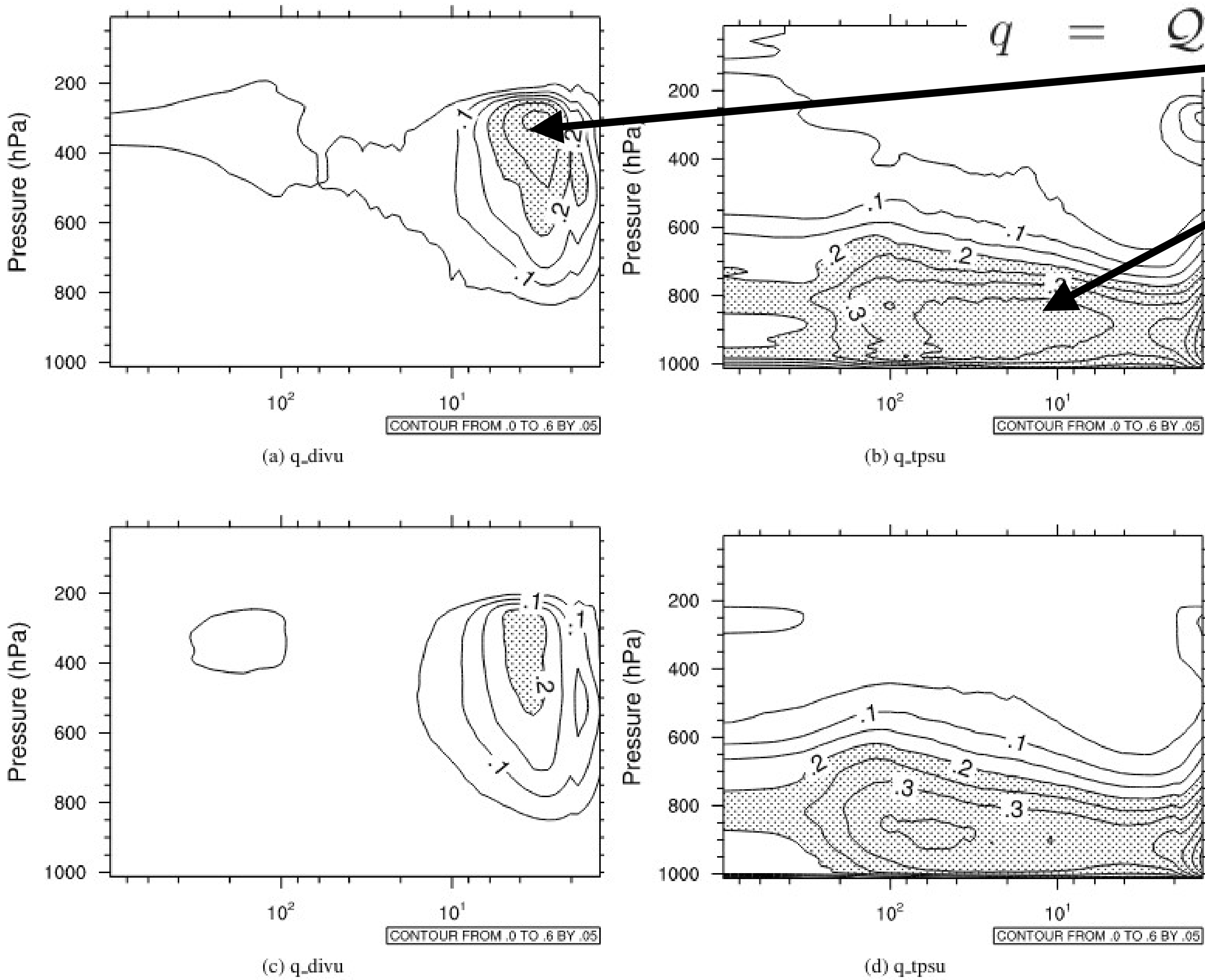
  - climatological B : few members x numerous dates (numerous and different meteorological situations sampled)

  For arome operational 1.3L90 -> 120 forecast differences :
  6 members x 10 winter days (00UTC)
  + 6 members x 10 summer days (12UTC for convection)

Question 1 : When computing a climatological B-matrix by an ensemble technique, how many members and how long should be the period to consider and why ?

- Yann Michel tried with a extended data set (400 members) : differences only on cross-correlations :

$$q = \mathcal{QH}\zeta + \mathcal{R}\eta_u + \mathcal{S}(T, P_s)_u + q_u$$

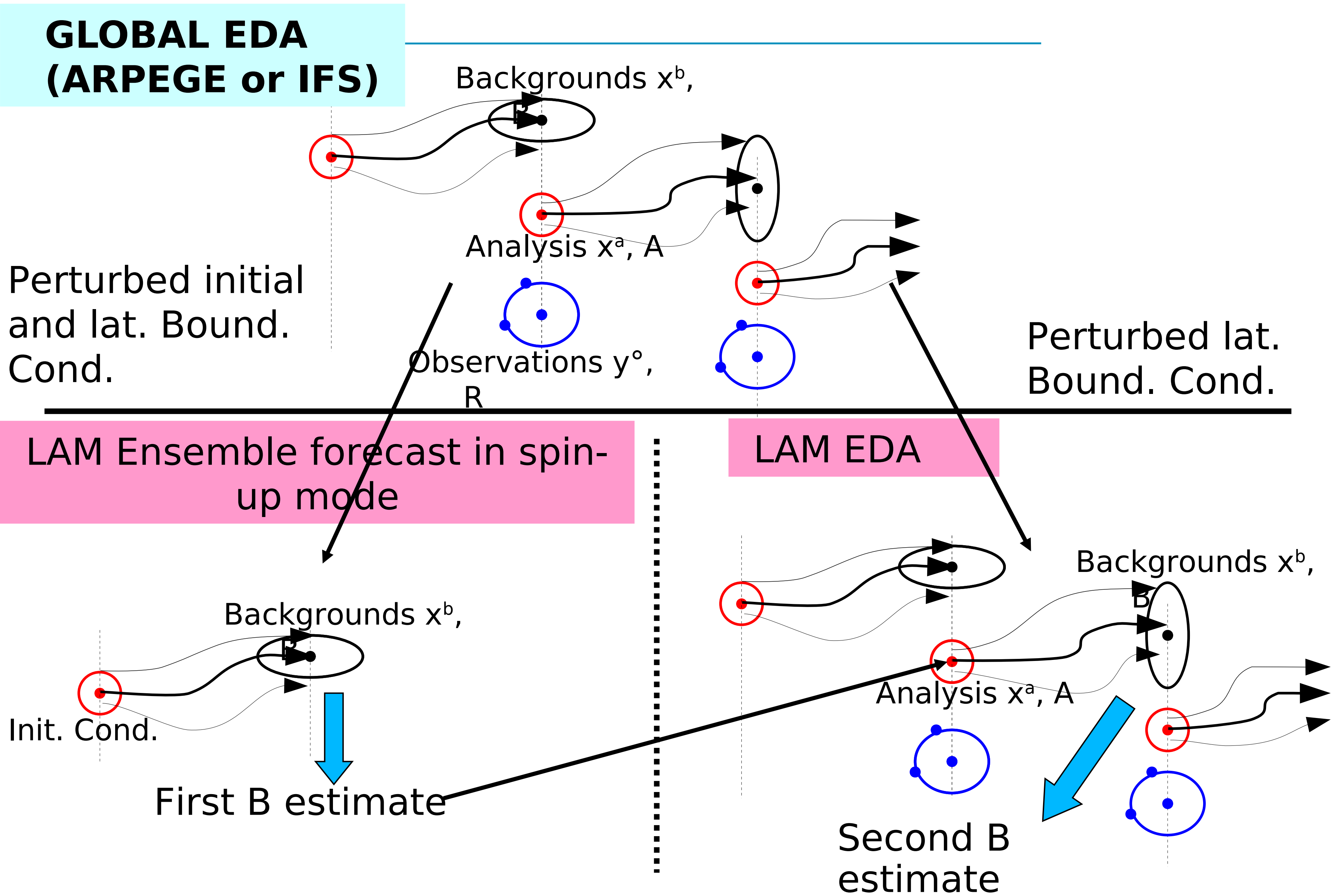


120 members

400 members

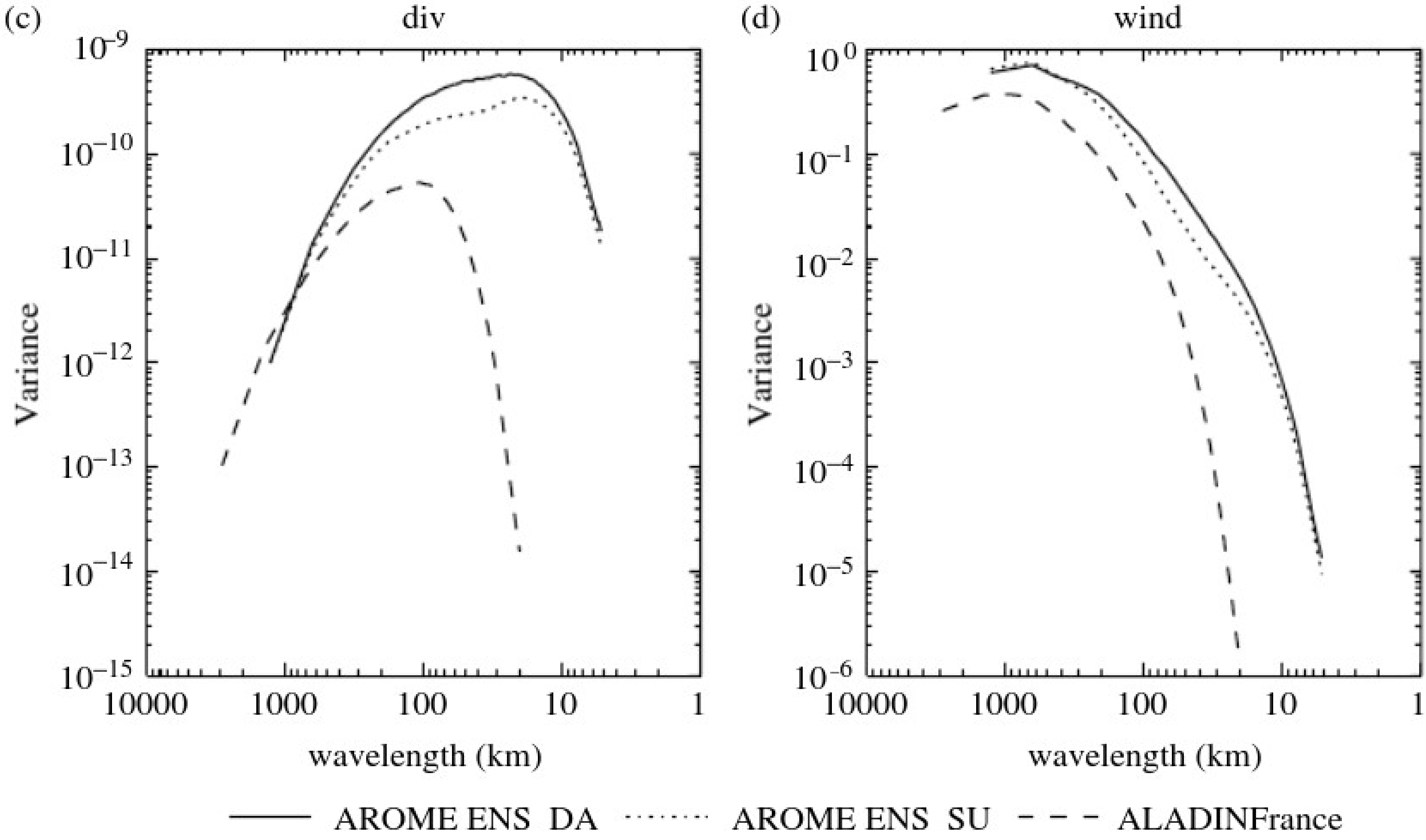
But no impact on the system performances…

Ensemble Data Assimilation



ENS-SU Vs ENS-DA

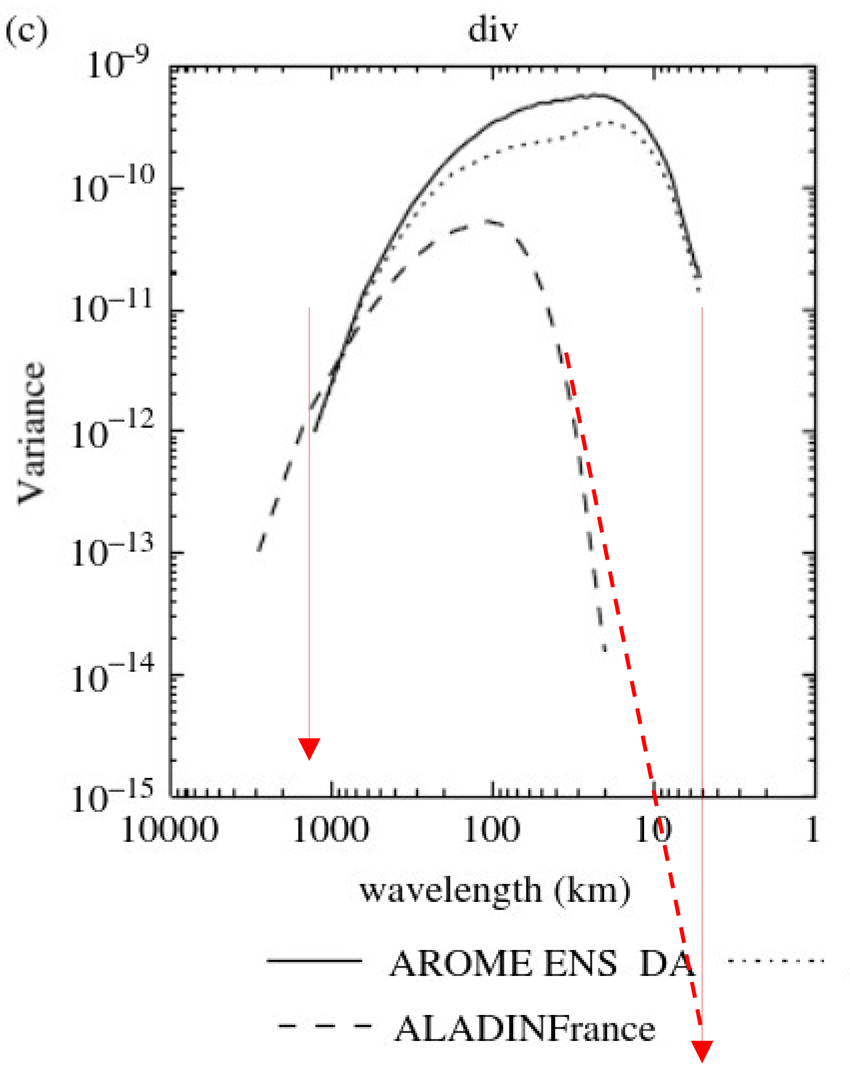
Variance spectra : stronger uncertainties for smaller scales



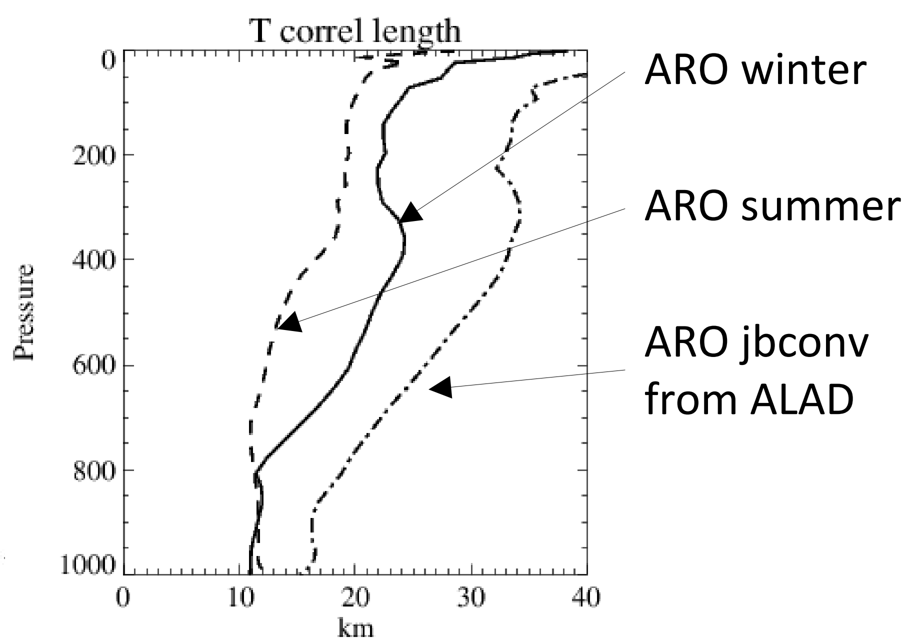
Ens. Spin-up mode provides a good proxi of the covariances obtained using a LAM EDA brousseau et al. 2011

Question 2 : is it possible to short-cut B-matrix computation: i) by cutting it from a geographically larger B-matrix; ii) from a lower resolution B-matrix;

Variance spectra : stronger uncertainties for smaller scales



JBCONV tool (F. bouttier) allows to transpose a larger domain/lower resolution spectra to a smaller/higher resolution one :
- but need to correct/extrapolate for the smaller wavelength leading to too long horizontal lenthscales



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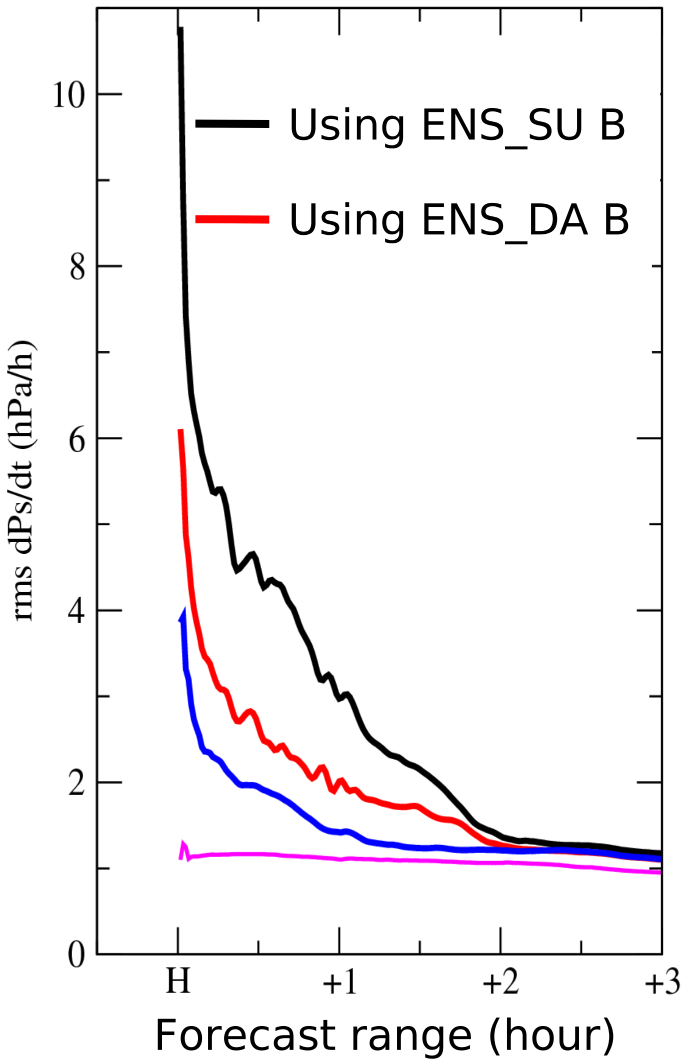
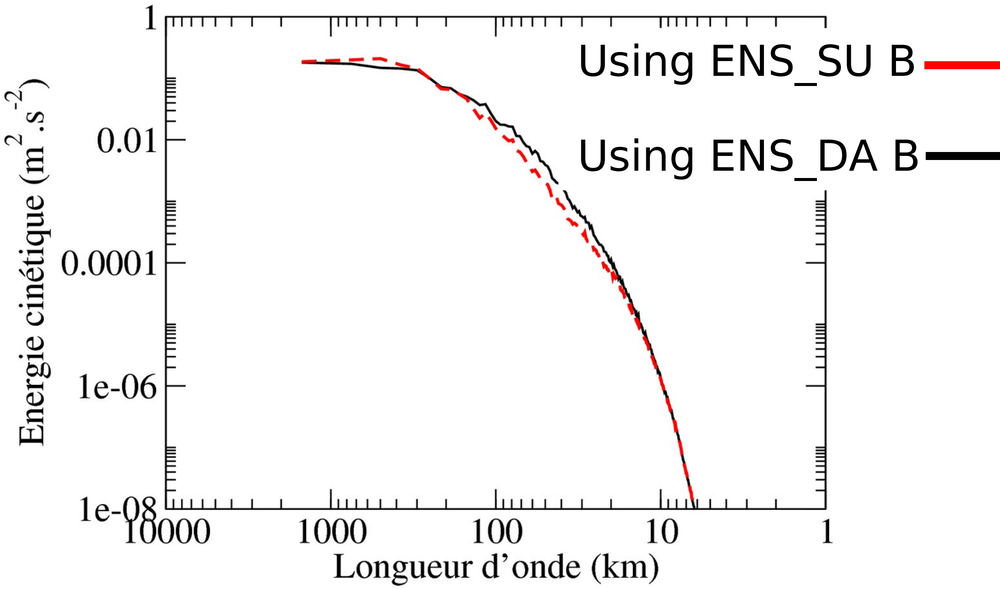


- **Spin-up weaker using ENS_DA B matrix**
- **Arome-france 1-h cycle is not possible using ENS_SU : too much spin-up** brousseau et al.2016
- **ENS_DA accentuates the small scales of the analysis increment**



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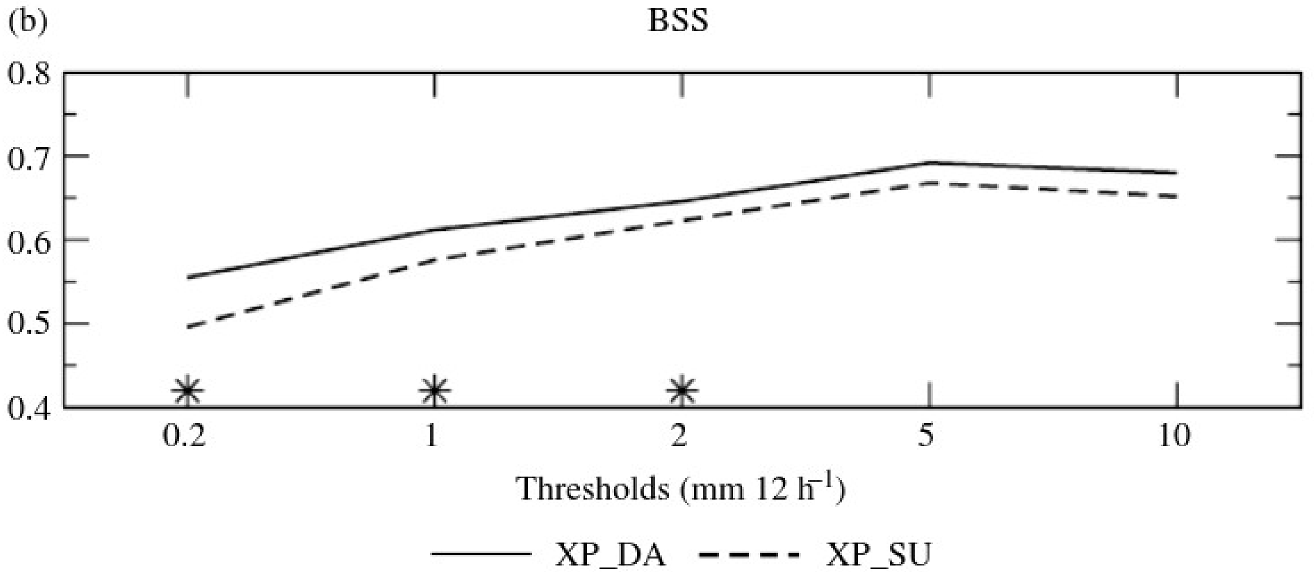
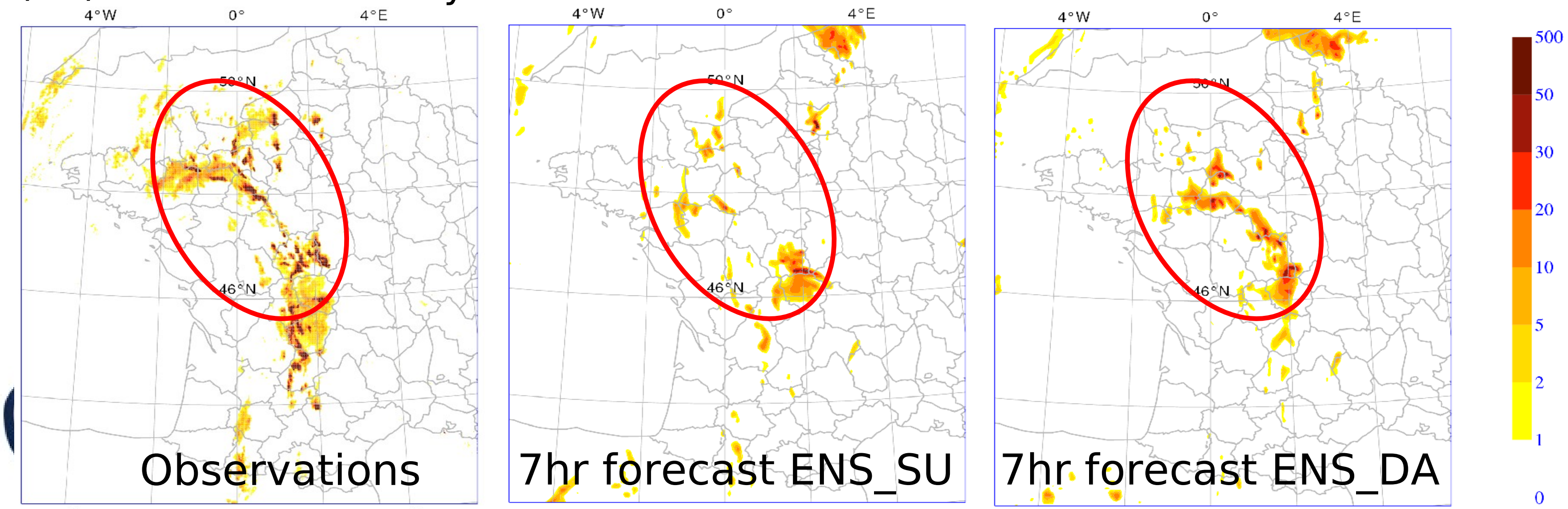


- Precipitation scores against raingauges for a 3 weeks period
- 25/05/2009 convective system : observed and simulated reflectivities at 19 UTC



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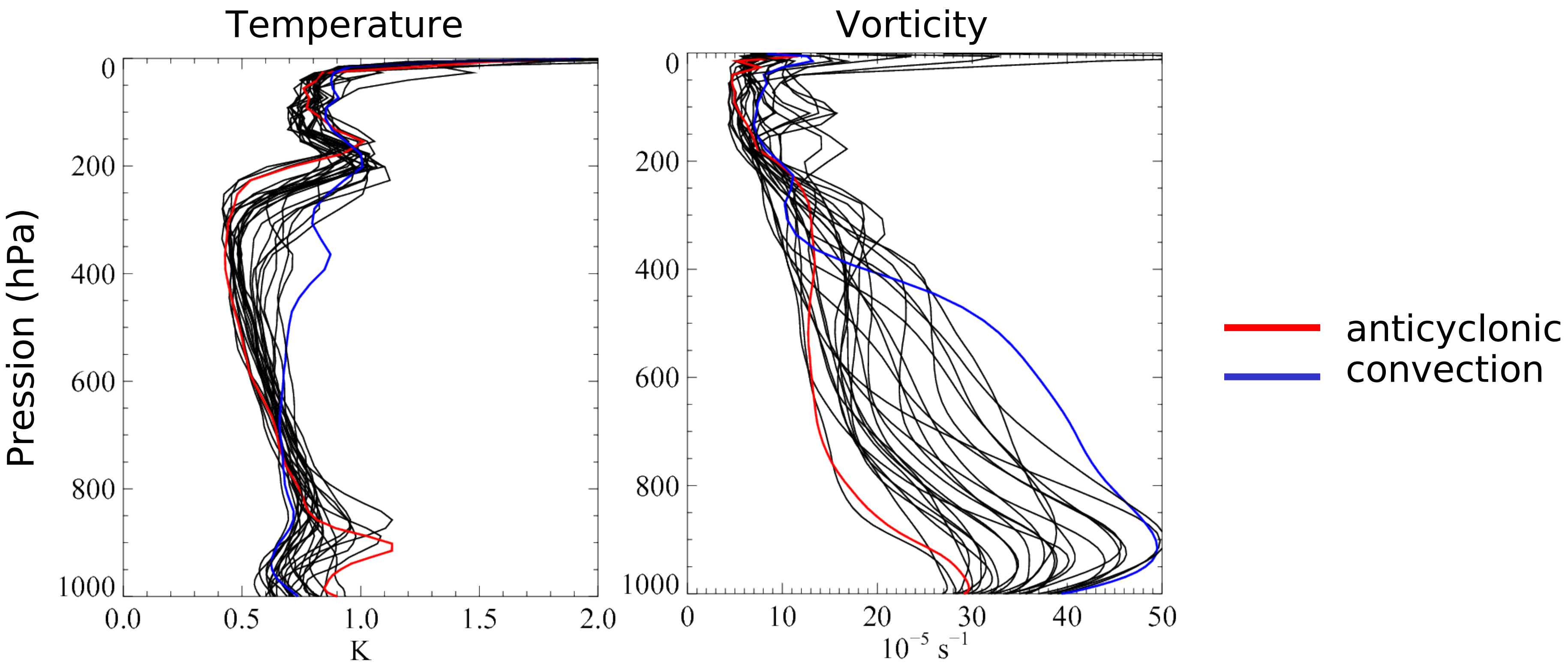
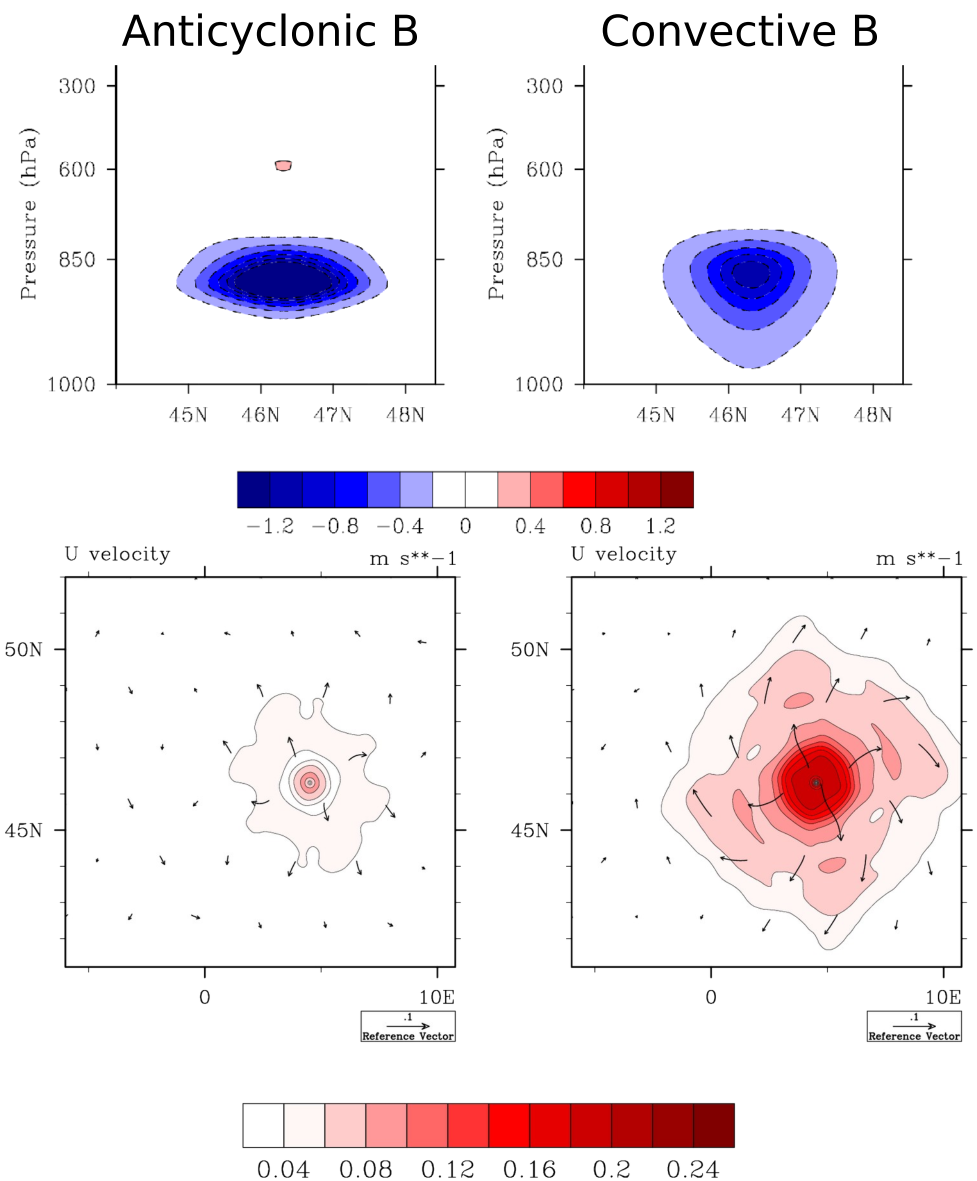
Question 8 : How much impact is expected from daily recalculation of B (in 3D-Var) if a real-time ensemble is available? Is it worth the effort or is it better to plan to set up the EnVar in this case?

- Homogeneity asumption : average over the domain
- Temporal average over a day : B varying daily



Daily vertical profiles of background error sigma-b

Question 8 : How much impact is expected from daily recalculation of B (in 3D-Var) if a real-time ensemble is available? Is it worth the effort or is it better to plan to set up the EnVar in this case?

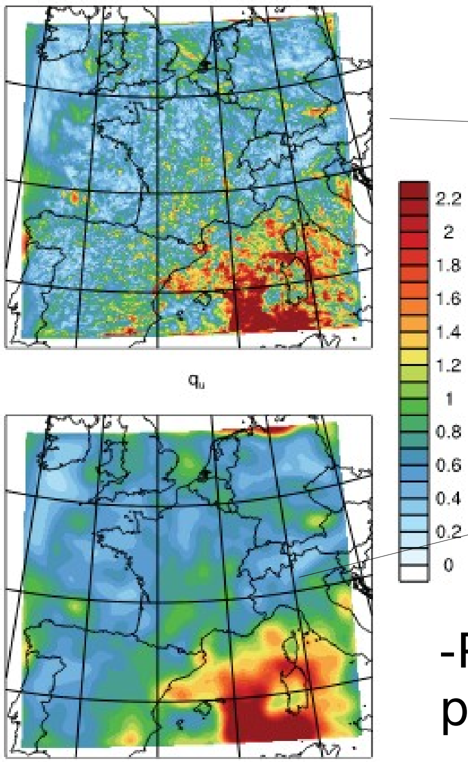


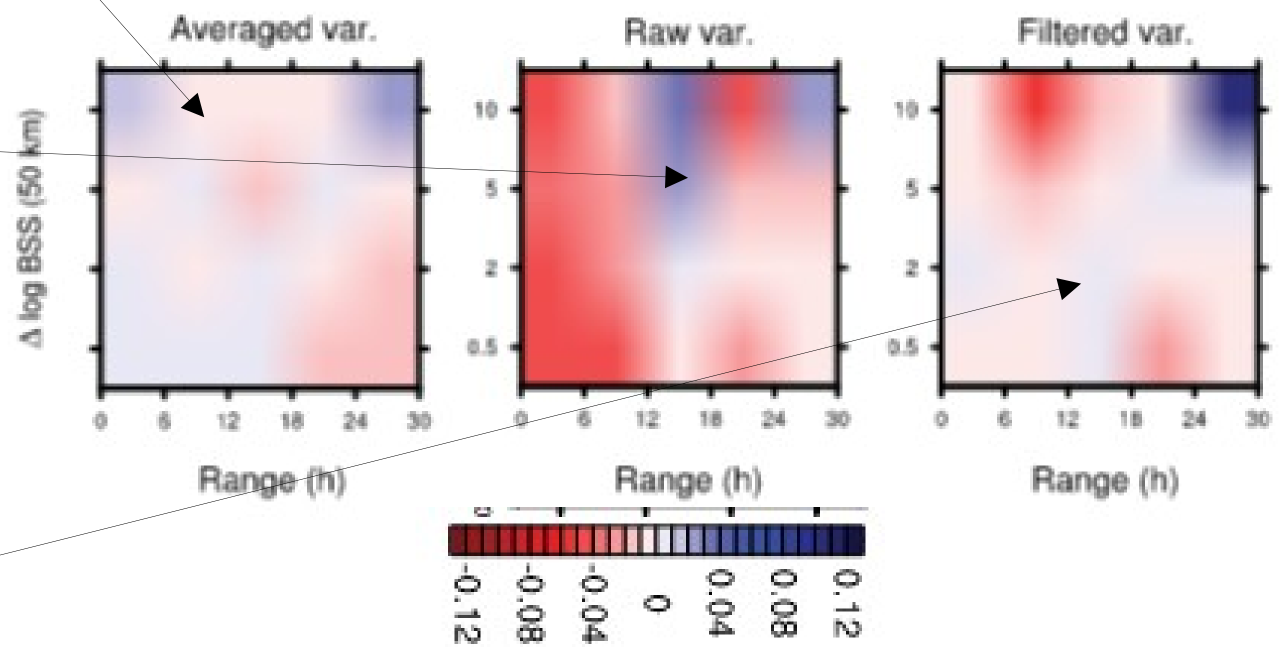
- Same innovation at 850 hPa
- Vertical cross section of the temperature analysis increment
- Horizontal cross section of the wind increment at 950 hPa

Question 8 : How much impact is expected from daily recalculation of B (in 3D-Var) if a real-time ensemble is available? Is it worth the effort or is it better to plan to set up the EnVar in this case?

- But very weak impact in arome on the forecast performances compared to ENS_DA Vs ENS_SU (brousseau et al. 2012)

- ... and negative impact of grid-point sigma-b (Benjamin Menetrier PhD)

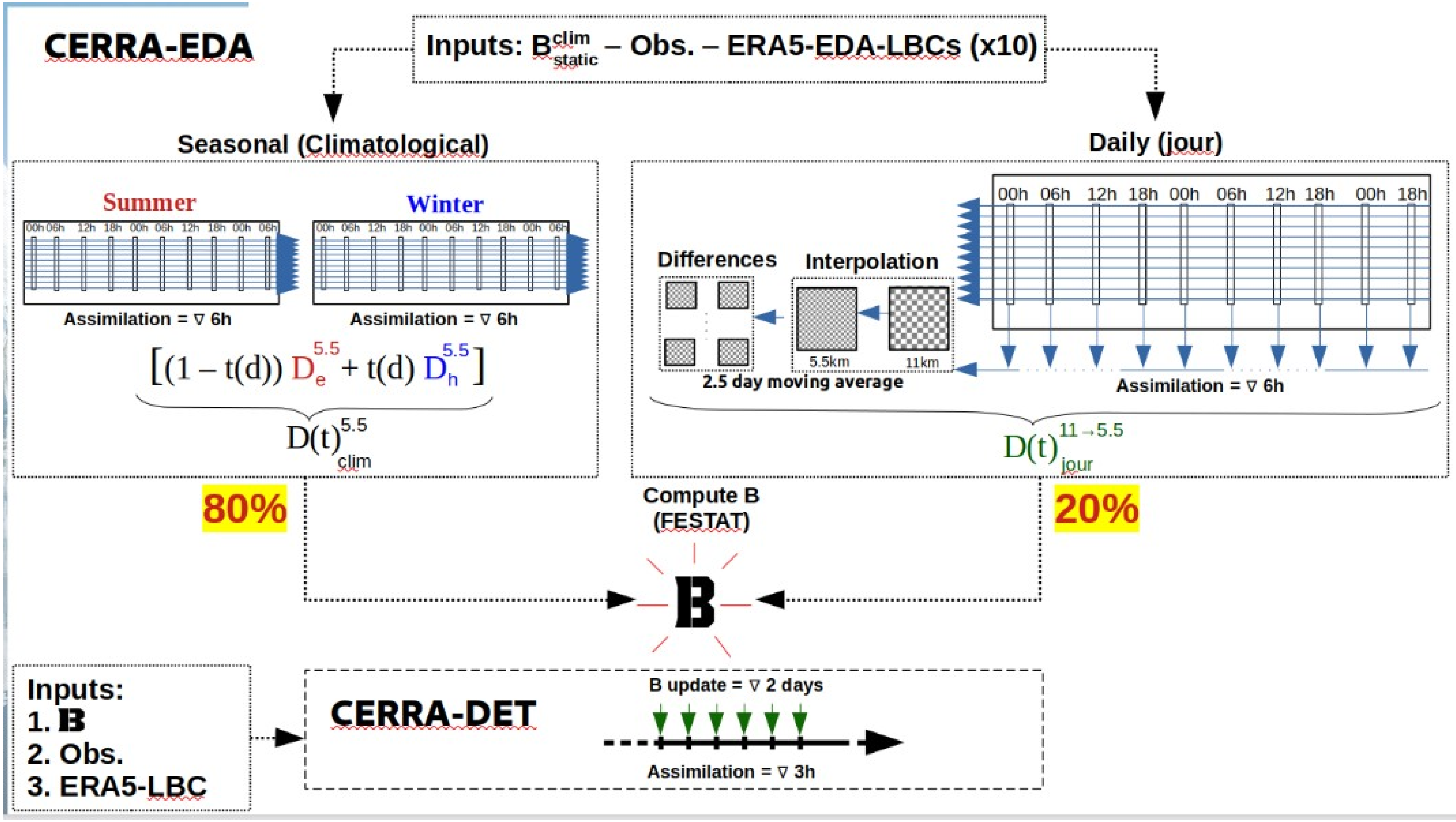




-Precipitation scores against raingauges for a 2 month period : improvement degradation

Question 8 : How much impact is expected from daily recalculation of B (in 3D-Var) if a real-time ensemble is available? Is it worth the effort or is it better to plan to set up the EnVar in this case?

- Slight positive impact in CERRA reanalysis (Adam El Said)



Question 7 : How important it is to use the same forecast length in the sampling of differences for B and in the actual DA cycling (e.g. forecasts of 3h length for 3-hourly cycle or 1h for hourly cycle)

- σb depend on the guess range and are expected to increase with this range

- for arome-france hourly cycle : estimated σb from the arome EDA at a 1 h forecast range are close to (and sometimes higher than) those obtained at 3 h forecast range, whereas smaller σb were expected.

- And diagnosed : using rmse and desroziers diagnostics in an iterative process leading to : σb1h/σb3h=0.5

Table 1. The first row shows the ratios of root mean square errors of 1 and 3 h range AROME-France forecasts for different observation types. The lower rows show ratios of background-error standard deviation of 1 and 3 h range backgrounds estimated in the observation space using Desroziers *et al.* (2005) during the iterative process. The last column shows averaged values over all obervation types.

Obs	T_{2m}	RH_{2m}	Gr. GPS	Rad. W	Rad. RH	Air. T	Air. W	SEVIRI T_B	IASI T_B	Average
$\frac{\text{rmse (1h)}}{\text{rmse (3h)}}$	0.89	0.77	0.81	0.87	0.8	0.83	0.78	0.83	0.77	0.81
α_{n+1} (α_n) for α_n										
1	0.67	0.69	0.80	0.76	0.74	0.66	0.76	0.74	0.85	0.74
0.75	0.55	0.58	0.67	0.63	0.63	0.54	0.64	0.62	0.70	0.61
0.6	0.48	0.48	0.49	0.51	0.47	0.45	0.52	0.55	0.64	0.51
0.5	0.47	0.49	0.48	0.52	0.45	0.43	0.51	0.56	0.60	0.50

- So, the arome-france hourly cycle uses a 3h forecast B matrix with REDNMC=0.5

Conclusion

→ B plays a key role in a DA system as it determines how the observations modify the background to build the analysis

→ B should depend on the model and its resolution, the geographical area, the meteorological situation, the observation network

→ due to the large size of the NWP system, B can not be explicitly written and stored. It is modelized

→ the true state of the atmosphere is unknown : proxi of background error are obtained from forecast differences :

- ensemble in spin-up mode can provides a first proxi of B
- EDA provides a “better” one

→ offline EDA => climatological B

online EDA => some parts of the modelized B can become flow dependent … but with slight impact on forecast performances ..

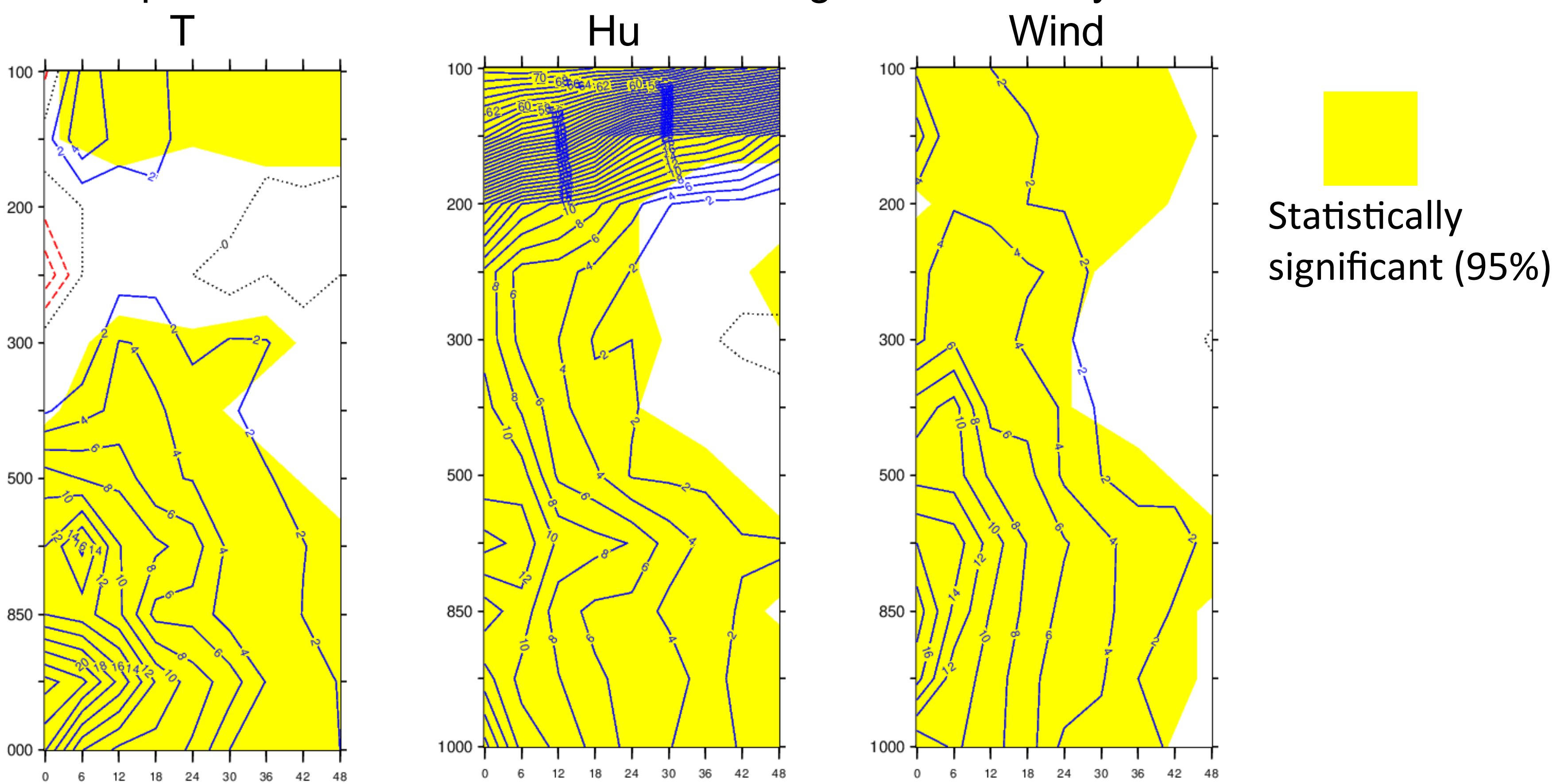
More promising results obtained from 3DEnvar ...

3DEnvar

→ B in no modelized but directly estimated from the ensemble perturbations **X** (~50 members) and localized to avoid long distance spurious correlations

3D-Var : $\mathrm{B} = \overline{\mathrm{B}} = \mathbf{K}\mathbf{B}_{\mathbf{u}}\mathbf{K}^{\mathrm{T}}$ → 3DEnVar : $\mathrm{B} = \widetilde{\mathrm{B}}_e = \mathbf{C} \circ \mathbf{X}\mathbf{X}^{\mathrm{T}}$

Relative improvement for 3DEnvar Vs 3DVar against IFS analysis :



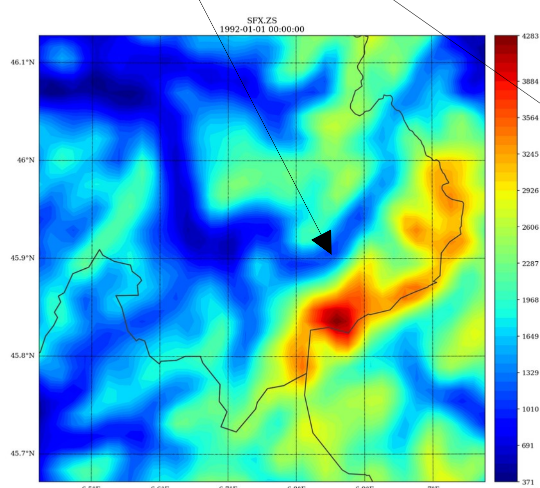
3DEnvar

→ B in no modelized but directly estimated from the ensemble perturbations **X** (~50 members) and localized to avoid long distance spurious correlations

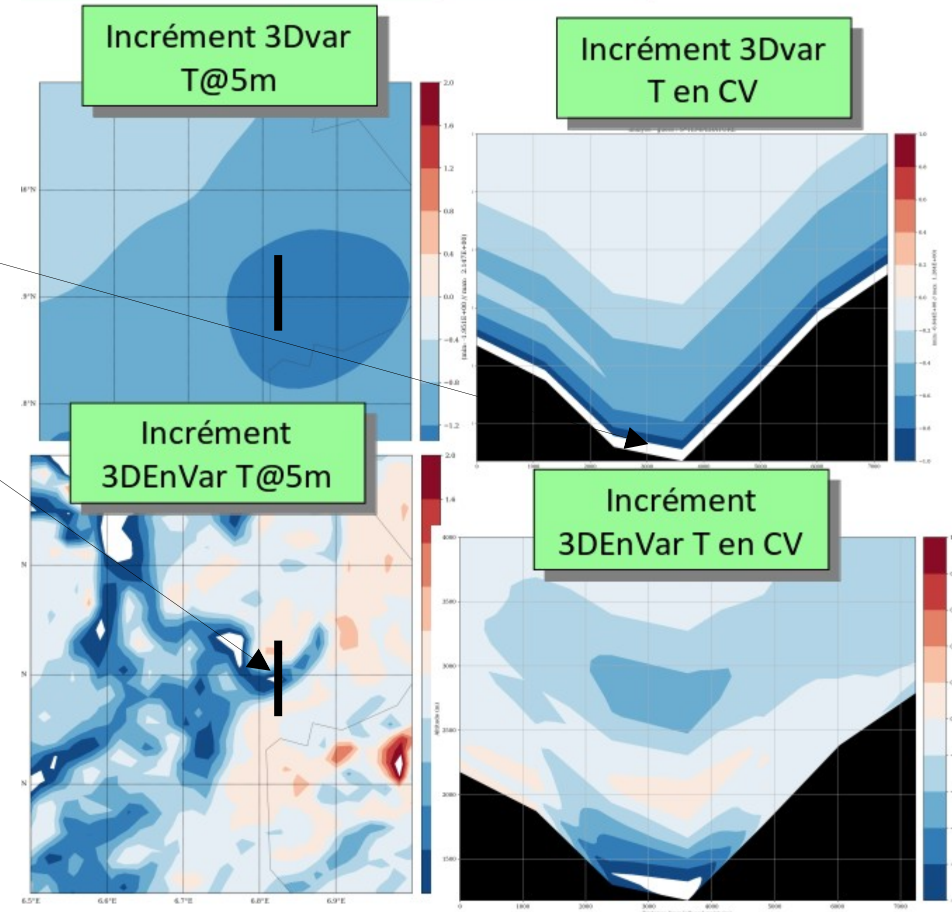
3D-Var : $\mathrm{B} = \overline{\mathrm{B}} = \mathbf{K B_u K^T}$ → 3DEnVar : $\mathrm{B} = \widetilde{\mathrm{B}}_e = \mathbf{C} \circ \mathbf{X X^T}$

Locally the increment can be seen as a linear combination of the perturbations : it is fully flow dependent

Chamonix valley in the Alps



Topography



Thank you for your attention...