TOUCANS and moist prognostic TKE and TTE

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TOUCANS

T - Third

O - Order moments (TOMs)

U - Unified

C - Condensation

A - Accounting and

N - N-dependent

S - Solver (for turbulence and diffusion)

Reynolds-averaged basic equations:

$$\frac{D\overline{u}}{dt} = S_u \boxed{-\frac{\partial \overline{u'w'}}{\partial z}}$$

$$\frac{D\overline{v}}{dt} = S_v \boxed{-\frac{\partial \overline{v'w'}}{\partial z}}$$

$$\frac{D\overline{s_{sL}}}{dt} = S_{s_{sL}} \boxed{-\frac{\partial \overline{s_{sL}'w'}}{\partial z}}$$

$$\frac{D\overline{q_t}}{dt} = S_{q_t} \boxed{-\frac{\partial \overline{q_t'w'}}{\partial z}}$$

(u, v, w -wind components, S_x - external source terms, $\frac{D()}{dt} = \frac{\partial()}{\partial t} + \overline{u}\frac{\partial()}{\partial x} + \overline{v}\frac{\partial()}{\partial y}$, $\overline{()}$ - average, $()'$ - fluctuation ,z - height, t - time)

Diffused variables

- u, v - horizontal wind components
- q_t -total specific moisture
- $s_{sL} = c_{pd} \left(1 + \left[\frac{c_{pv}}{c_{pd}} - 1\right] q_t\right) T + g\, z - (L_v\, q_l + L_s\, q_i)$ - moist static energy

$q_{l/i}$ - specific humidity of air for liquid or ice water, $L_{s/v}$ latent heat latent heat of sublimation/vaporization, g - acceleration of gravity, T - air temperature, c_{pd} and c_{pv} specific heat values for dry air and water vapour

Turbulent fluxes

- $\overline{w'u'} = -K_M \frac{\partial \bar{u}}{\partial z}$,
  $\overline{w'v'} = -K_M \frac{\partial \bar{v}}{\partial z}$
- $\overline{w' s_{sL}'} = -K_H \frac{\partial \bar{s}_{sL}}{\partial z} + \mathrm{TOMs\ terms}$,
  $\overline{w' q_t'} = -K_H \frac{\partial \bar{q}_t}{\partial z} + \mathrm{TOMs\ terms}$
- $\overline{w'\psi'} = C_\psi \sqrt{u^2 + v^2} (\psi - \psi_s)$ - surface layer

$K_{M/H}$ - turbulent exchange coefficients for momentum and heat and moisture, C_ψ - drag coefficient, ψ - diffused variable, $()_s$ - variable at surface layer

Exchange coefficients

$$K_M = \frac{\nu^4}{C_\epsilon}\chi_3(\Pi)\sqrt{e_k}L, \quad K_H = C_3\frac{\nu^4}{C_\epsilon}\phi_3(\Pi)\sqrt{e_k}L$$

free parameters stab. functions	e_k, e_t $\Pi = e_t/e_k - 1$	length scale
given by turbulence scheme	prognostic turbulence energies	quasi-independent, may depend on TKE and BVF

$\chi_3(Ri_f)$,$\phi_3(Ri_f)$ - stability functions, ν - free parameter, C_3 - inverse Prantl number at neutrality, L - length scale

Framework of stability dependency functions:

- based on second order moments equations
- simple and flexible emulation of variety of turbulent schemes:
  - comparison of schemes
  - physics ensemble modeling
- properties of χ_3,ϕ_3 (Bašták, Geleyn, and Váňa, 2014):
  - valid for whole range of Ri
  - no existence of critical Ri - Ri_{cr}
  - anisotropy of turbulence:
    - $\frac{\partial \chi_3}{\partial Ri} \neq 0$, $\frac{\partial \phi_Q}{\partial Ri} \neq 0$

(Ri -gradient Richardson number, ϕ_Q - non - energy conversion part of ϕ_3 - coefficient)

Framework of stability dependency functions:

- simple shape in terms of Ri_f:

$$\chi_3(Ri) = \frac{1-\frac{Ri_f}{R}}{1-Ri_f} \quad , \phi_3(Ri) = \frac{1-\frac{Ri_f}{P}}{1-Ri_f} \quad ,$$

$$\phi_Q(Ri) = \frac{1-\frac{Ri_f}{Q}}{1-Ri_f} \quad , \frac{Ri}{Ri_f} = \frac{P(R-Ri_f)}{C_3\, R\,(P-Ri_f)}$$

$$0 < \lim_{Ri\to\infty} P = Ri_{fc} < 1,\ Ri_{fc} < \lim_{Ri\to\infty} R \equiv R_\infty \leq 1,\ Ri_{fc} \leq \lim_{Ri\to\infty} Q \equiv Q_\infty \leq 1.$$

- factorization of $\phi_3(Ri)$:

$$\phi_3 = \underbrace{\phi_Q(Ri)}_{\text{anisotropy}} \underbrace{\left(1 - \frac{2\,O_\lambda\ (e_t - e_k)}{C_4\,\overline{w'^2}}\right)}_{\text{energy conversion}}, \frac{\partial \phi_Q}{\partial Ri} \neq 0$$

($Ri_f = RiK_H/K_M$ -flux Richardson number, O_λ -free parameter, C_4 - coefficient)

Framework of stability functions:

- the turbulent scheme then depends on:
  - 4(3) free parameters
    - ν - overall turbulence intensity,
    - C_ϵ - turbulent energy dissipation
      - following Schmidt and Schumann (1989)
      we assume: $C_\epsilon = \pi \, \nu^2$,
    - C_3 -inverse Prandtl number at neutrality,
    - O_λ - TKE$\leftrightarrow$TPE conversion,
  - 3 "functional dependencies" (P, R, Q)

	Model I	Model II	eeQNSE	eeEFB
P	Const.	Const.	Const.	Ri fun.
R	Const.	Const.	Ri fun.	Ri fun.
Q	Const.	Ri fun.	Ri fun.	Ri fun.

Emulation and extension of turbulent schemes:

- turbulent schemes without Ri_{cr} can be emulated in BGV2014 framework
- continuous extension to unstable regime ($Ri < 0$) is required for schemes that are defined only in stable regime($Ri > 0$)
- eeQNSE = emulation and extension of Quasi Normal Scale Elimination (QNSE) scheme - Sukoriansky et al. (2005)
- eeEFB = emulation and extension of Energy- and Flux-Budget (EFB) scheme - Zilitinkevich et al. (2013)

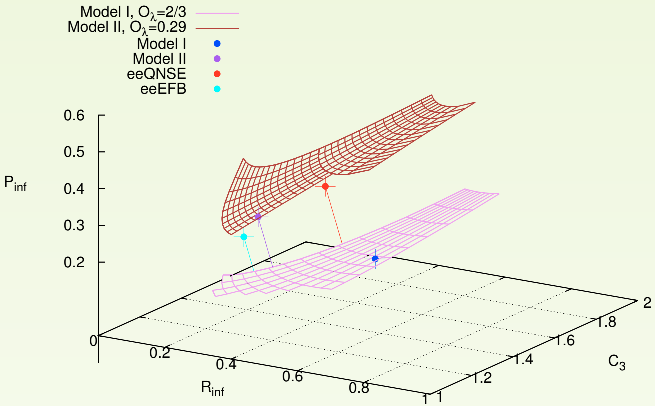
Comparison of schemes :

Free parameters:

	Model I	Model II	eeQNSE I	eeQNSE II	eeEFB
ν	0.526	0.526	0.464	0.504	0.532
C_3	1.18	1.18	1.39	1.39	1.25
O_λ	2/3	0.290	0.248	0.324	0.113
C_ϵ	0.871	0.871	0.676	0.798	0.889

(eeQNSE I - emulation and extension of QNSE in Model I, eeQNSE II - emulation and extension of QNSE in Model II, eeEFB - emulation and extension of EFB in Model I)

Comparison of schemes :

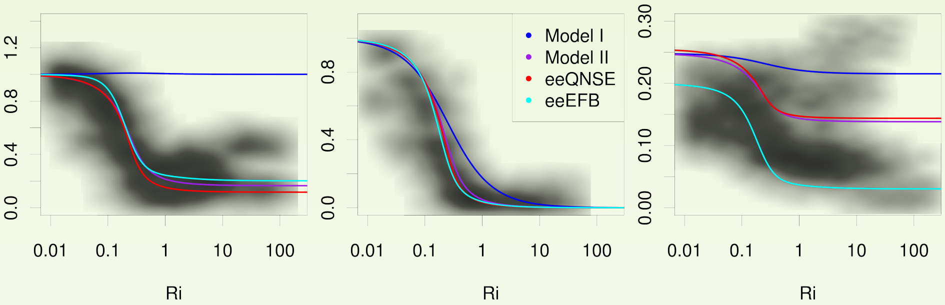


Comparison of schemes:

Scaled dimensionless squared momentum flux

Scaled dimensionless squared heat flux

A_z



(data aggregating laboratory experiments (Ohya 2001), LES (Zilitinkevich et al. 2007, 2008), DNS(Stretch et al. 2001), and meteorological observations (Mahrt and Vickers 2005; Uttal et al. 2002; Poulos et al. 2002; Banta et al. 2002; Engelbart et al. 2000))

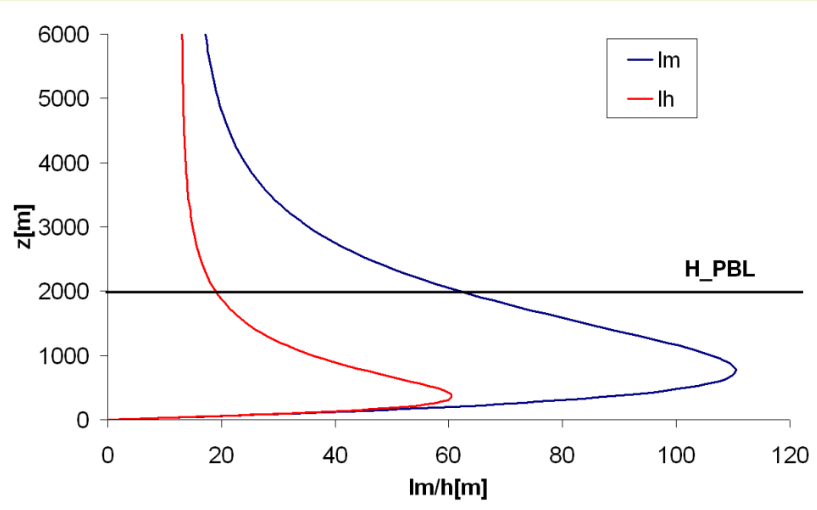
Prandtl-type mixing lengths l_m and l_h

(CGMIXLEN='AY', in ALARO0='CG') :

$$l_{m/h}^{GC} = \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_{m/h}} \left[\frac{1+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)}{\beta_{m/h}+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)} \right]}$$

(κ is Von Kármán constant, z is height, $a_{m/h}$, $b_{m/h}$, $\beta_{m/h}$ and $\lambda_{m/h}$ are tuning constants and H_{pbl} is PBL height)

Prandtl-type mixing lengths:



TKE based length scales L

- Bougeault a Lacarrère (1989) :

$$L_{BL}(E) = \left(\frac{L_{up}^{-\frac{4}{5}} + L_{down}^{-\frac{4}{5}}}{2} \right)^{-\frac{5}{4}}$$

$L_{up}(E)$ ($L_{down}(E)$) - L upward (downward)

- $L_N = \sqrt{\frac{2.E}{N^2}}$ for stable stratification
- with possibility to use moist BVF
- possible prognostic treatment of L

Conversion between L and l_m

- following RMC01:

$$L_K = \frac{C_\epsilon}{\nu^3} l_m \frac{f(Ri)^{\frac{1}{4}}}{\chi_3^{\frac{1}{2}}}, \quad L_\epsilon = \frac{C_\epsilon}{\nu^3} l_m \frac{\chi_3^{\frac{3}{2}}}{f(Ri)^{\frac{3}{4}}}$$

- assuming: $L \equiv \left(L_K^3 L_\epsilon\right)^{\frac{1}{4}}$ we get:

$$L = \frac{C_\epsilon}{\nu^3} l_m$$

Third Order Moments (TOMs)

- parametrization for heat and moisture
- following (Canuto, Cheng, and Howard, 2007):

$$\boxed{\overline{w'\theta'} = -K_H \frac{\partial\overline{\theta}}{\partial z}} + \boxed{A_1^\theta \frac{\partial\overline{w'^3}}{\partial z} + A_2^\theta \frac{\partial\overline{w'\theta'^2}}{\partial z} + A_3^\theta \frac{\partial\overline{w'^2\theta}}{\partial z}}$$

$$\overline{w'^3} = -0.06\frac{g}{\theta}\tau^2\overline{w'^2}\frac{\partial\overline{w'\theta'}}{\partial z}, \quad \overline{w'\theta'^2} = -\tau\overline{w'\theta'}\frac{\partial\overline{w'\theta'}}{\partial z}, \quad \overline{w'^2\theta'} = -0.3\tau\overline{w'^2}\frac{\partial\overline{w'\theta'}}{\partial z}$$

A_1^θ, A_2^θ, A_3^θ - coefficients, τ - dissipation time scale

- two step solver: local + non-local correction

Prognostic TTE/TPE

- based on Zilitinkevich et al.(2013)
- addition of second prognostic turbulent energy: Turbulent Potential Energy (TPE), or TTE = TKE+TPE
- consideration of counter-gradient heat transport maintained by velocity shear in very stable stratification
- stability parameter based on energy ratio Π =TPE/TKE (linked to fluxes) rather than on local gradients (Ri)

Prognostic TKE-e_k, TPE-e_p eqs.:

- based on Zilitinkevich et al.(2013)

$$\frac{d\,e_k}{dt} = -g\,\frac{\partial}{\partial p}\left(\rho\,K_{e_k}\frac{\partial e_k}{\partial z}\right) + I + II - \frac{2\,e_k}{\tau_k}$$

$$\frac{d\,e_p}{dt} = -g\,\frac{\partial}{\partial p}\left(\rho\,K_{e_p}\frac{\partial e_p}{\partial z}\right) - II - \frac{2\,e_p}{\tau_p}$$

$$\tau_p \equiv \tau_k\frac{C_4}{2\,C_3}, \quad I = -\overline{u'w'}\frac{\partial u}{\partial z} - \overline{v'w'}\frac{\partial v}{\partial z}$$

(I, II - shear and buoyancy source terms, K_{e_k}, K_{e_t} - turbulent exchange coefficients for TKE and TTE, τ_k, τ_p, τ_t - dissipation time scale for TKE, TPE and TTE, Π - stability parameter, C_3 - inverse Prandtl number at neutrality, C_4 - coefficient, p - pressure, g - acceleration of gravity, ρ - density)

Prognostic TTE - $e_t = e_k + e_p$ equation:

$$
\begin{aligned}
\frac{de_t}{dt} &= -g\frac{\partial}{\partial p}\left(\rho\, K_{e_t}\frac{\partial e_t}{\partial z}\right) + I - \frac{2\, e_t}{\tau_t} \\
\tau_t &\equiv \tau_k \frac{C_4\,(1+\Pi)}{C_4 + 2\,C_3 \Pi}, \\
Ri_f &= \frac{\Pi}{\frac{C_4}{2\,C_3} + \Pi}
\end{aligned}
$$

(e_p - TPE, K_{e_t} - turbulent exchange coefficient for TTE, τ_t - dissipation time scale for TTE, $Ri_f = Ri\, K_H/K_M$ - flux Richardson number, C_4 - coefficient)

Dry versus moist case:

- dry:

$$II_d = \frac{g}{\theta}\overline{\theta' w'}, \quad e_{pd} = \frac{g}{\theta}\frac{\overline{\theta'^2}}{2\frac{\partial\theta}{\partial z}}$$

- moist:

$$II_m = \frac{g}{\rho_0}\overline{w'\rho'} = E_{s_{sL}}\overline{w' s_{sL}'} + E_{q_t}\overline{w' q_t'}$$

$$e_{pm} = \frac{E_{sL}\overline{s_{sL}'2}}{2\frac{\partial s_{sL}}{\partial z}} + \frac{E_{q_t}\overline{q_t'^2}}{2\frac{\partial q_t}{\partial z}}.$$

- E_{sL}, E_{q_t} are derived after (Marquet and Geleyn, 2013) and depend on cloud fraction C and skewness parameter C_n

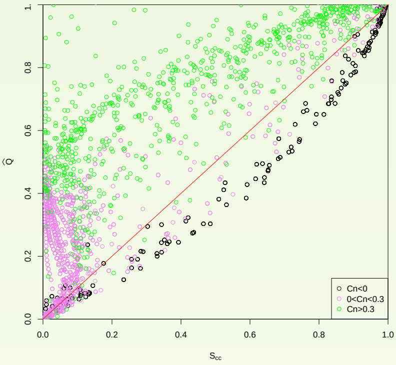
$$
\begin{aligned}
E_{s_{sL}} &= \frac{g\,M(C)}{\overline{c_p}\,\overline{\overline{T}}},\\
E_{q_t} &= g\,M(C)\Big\{\left(\frac{R_v - R_d}{R_d.\overline{q_d} + R_v.\overline{q_v}} - \frac{c_{pv} - c_{pd}}{\overline{c_p}}\right)\\
&\quad + \widehat{Q}\left[\frac{L_{vs}(\overline{T})(R_d.\overline{q_d} + R_v.\overline{q_v})}{\overline{c_p}\,\overline{\overline{T}}R_v} - 1\right]\cdot\\
&\quad \left[\frac{R_v - R_d}{R_d.\overline{q_d} + R_v.\overline{q_v}} + \frac{1}{(1-q_t)(1+D_C)}\right]\Big\}
\end{aligned}
$$

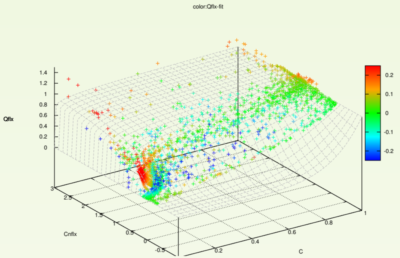
$\left(M(C) = \frac{1+D_C}{1+D_C\left(1+C\left[\frac{L_{vs}(\overline{T})(R_d.\overline{q_d}+R_v.\overline{q_v})}{\overline{c_p}\,\overline{T}R_v}-1\right]\right)}, D_C = \frac{L_{vs}(\overline{T})\,\overline{r_s^{li}}}{R_d\,\overline{T}} = \frac{\overline{T}}{\overline{p}-e_{sat}(\overline{T})}\frac{\partial e_{sat}(\overline{T})}{\partial\overline{T}} \right)$

$$\widehat{Q} = C^{F(C_n)}, \quad F(C_n) = 0.5\left[\sqrt{(6.25\,C_n)^2 + 4} - 6.25\,C_n\right]$$

$$C_n = \frac{-\frac{\overline{w's'_{sL}}}{\widehat{c_p}\,T} - \left(\frac{R_v - R_d}{R_d.\widehat{q_d} + R_v.\widehat{q_v}} - \frac{c_{pv} - c_{pd}}{\widehat{c_p}}\right)\overline{w'q'_t}}{\left[\frac{L_{vs}(\widehat{T})(R_d.\widehat{q_d} + R_v.\widehat{q_v})}{\widehat{c_p}\,T R_v} - 1\right]\left[\frac{R_v - R_d}{R_d.\widehat{q_d} + R_v.\widehat{q_v}} + \frac{1}{(1 - \widehat{q_t})(1 + D_C)}\right]\overline{w'q'_t}}$$

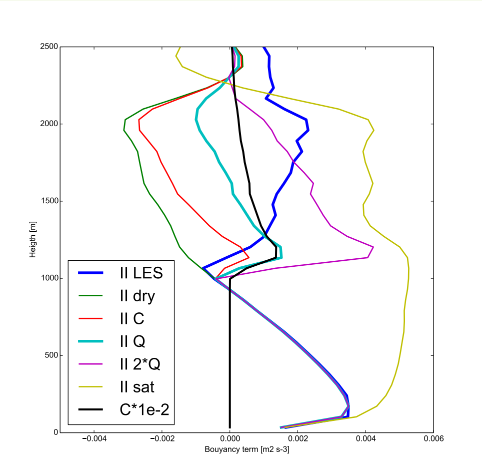
Fitting of $\widehat{Q}(C, C_n)$ on LES data (courtesy of D. Lewellen)





Buoyancy term *II*

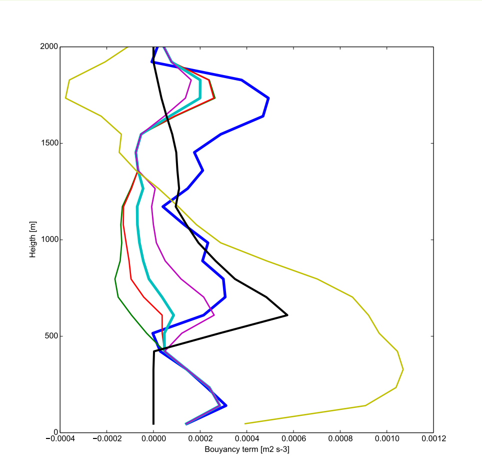
ARM case - Continental shallow cumulus:



LES data from microHH model (http://github.com/microhh).

Buoyancy term II

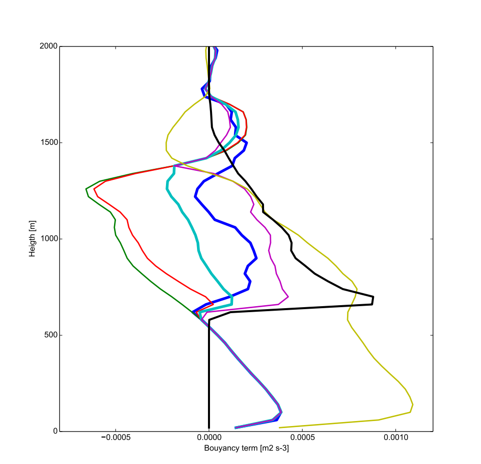
BOMEX case - Trade cumulus:



LES data from microHH model (http://github.com/microhh).

Buoyancy term II

RICO case - Precipitating trade cumulus:



LES data from microHH model (http://github.com/microhh).

Thank you for your attention