Convergence of the 3MT deep convection parameterization with the explicit convection at high resolution

Luc Gerard

February 2010

Topics

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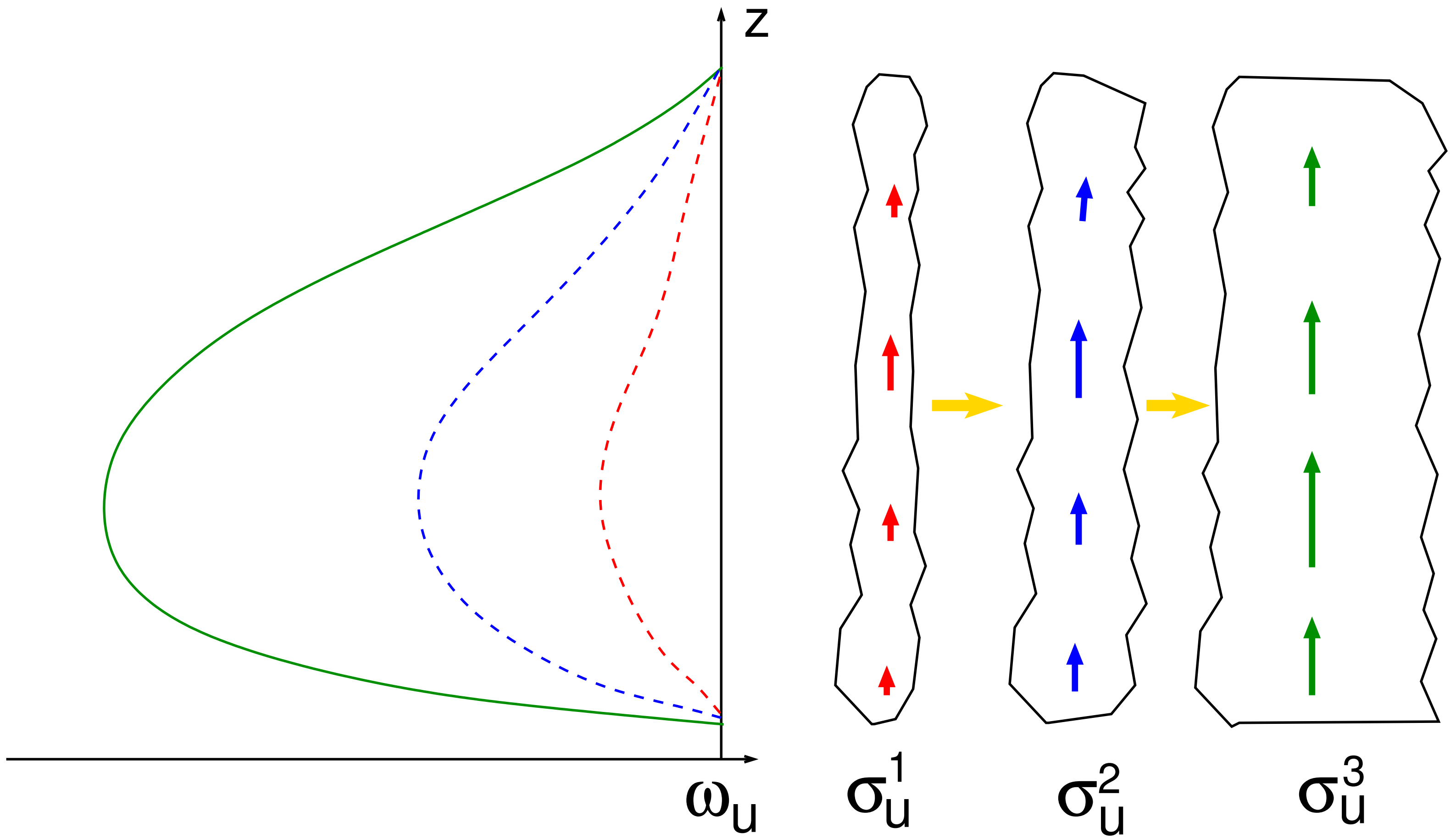
Part I : Cloud evolution.

Part II : Closure.

Part III : Preliminary Results.

The Alaro-0 vision of draft-evolution



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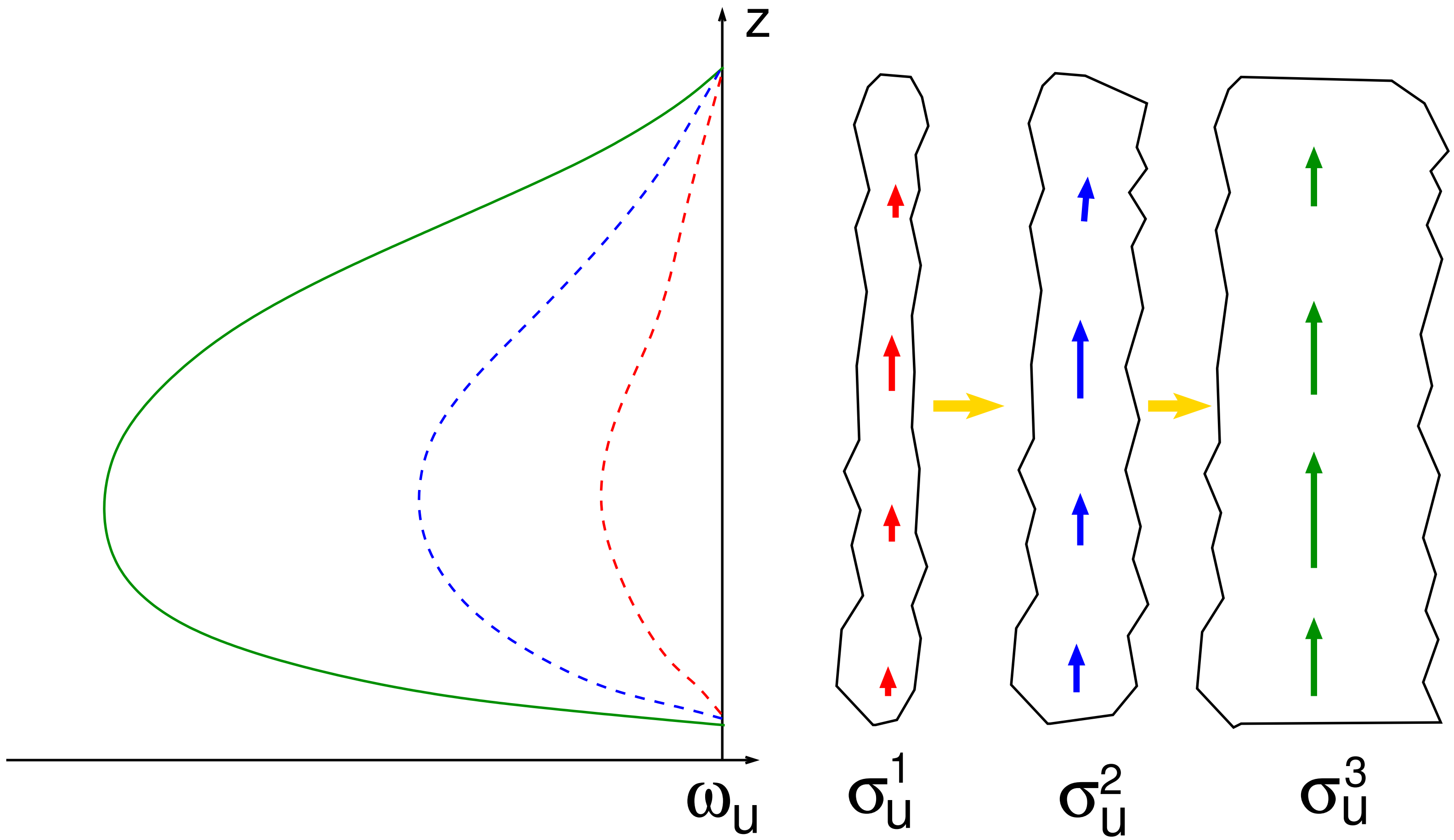
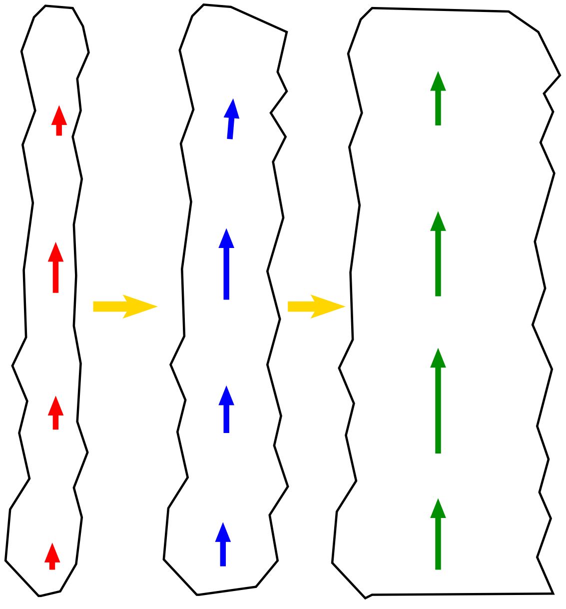


Instant-growth up to equilibrium level

Gradual increase of ω_u and σ_u

From Alaro-0 to nature to Alaro-1 concepts



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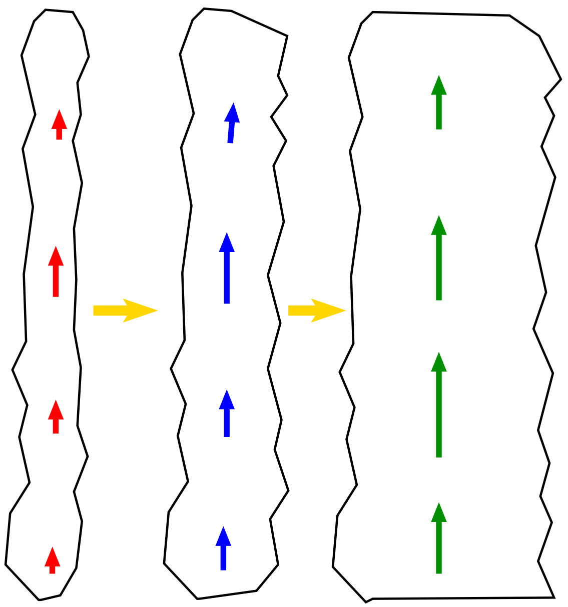

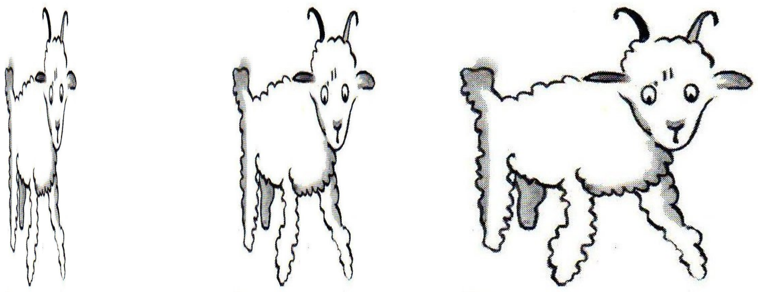

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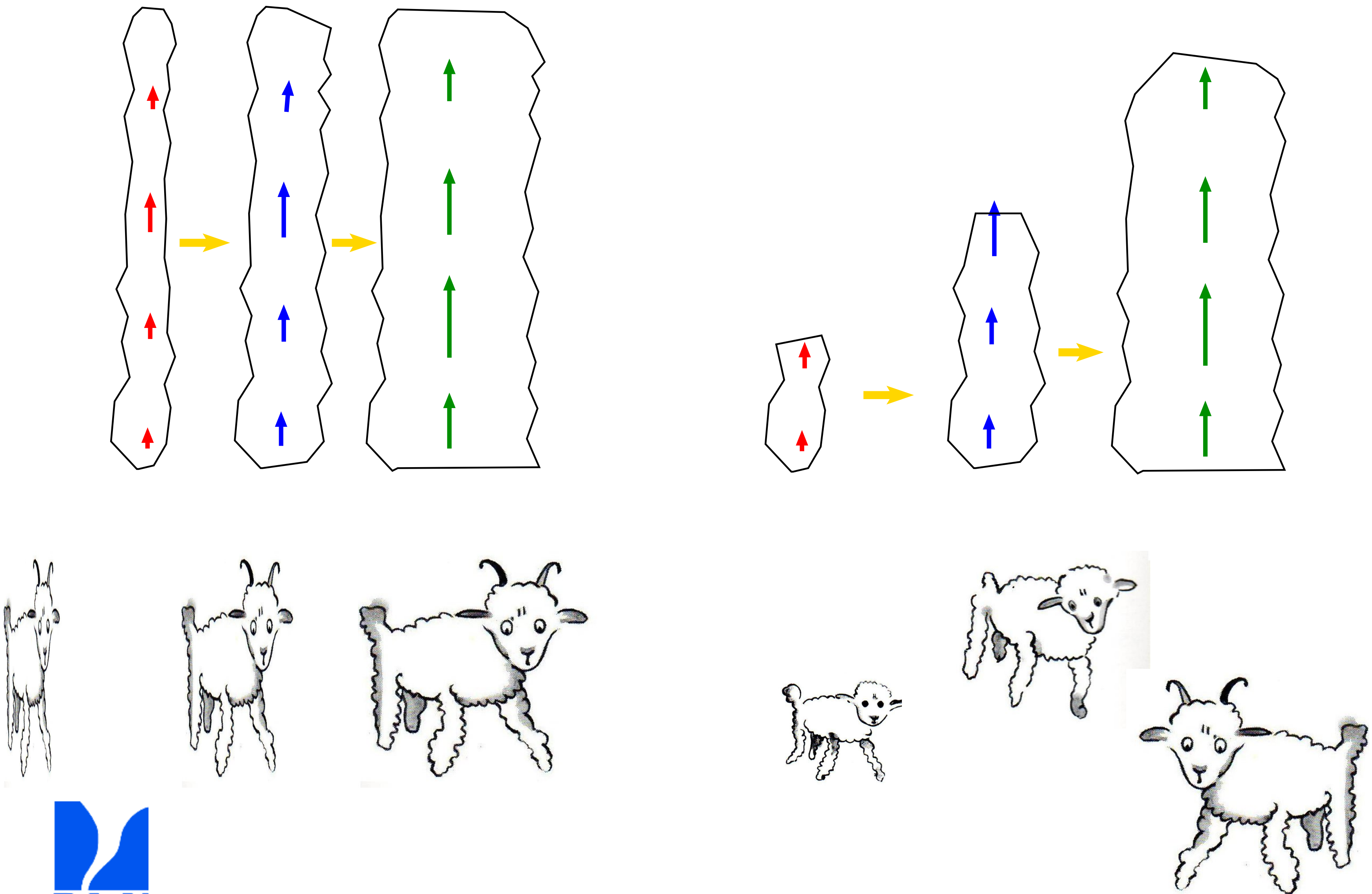

L. Gerard, February 2010

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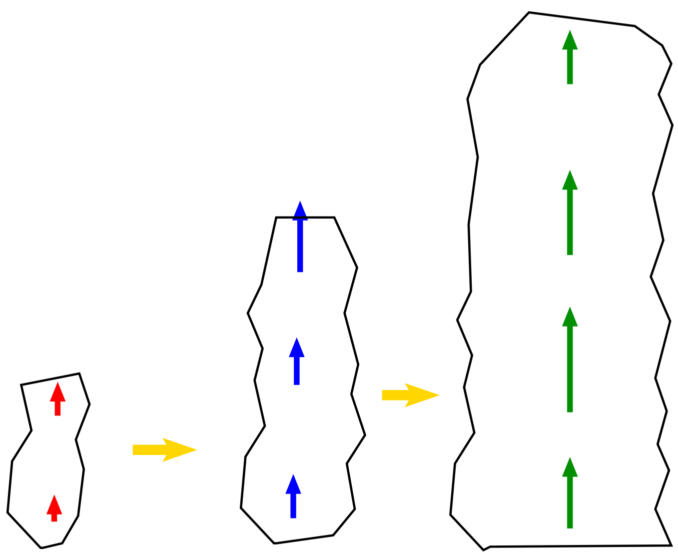

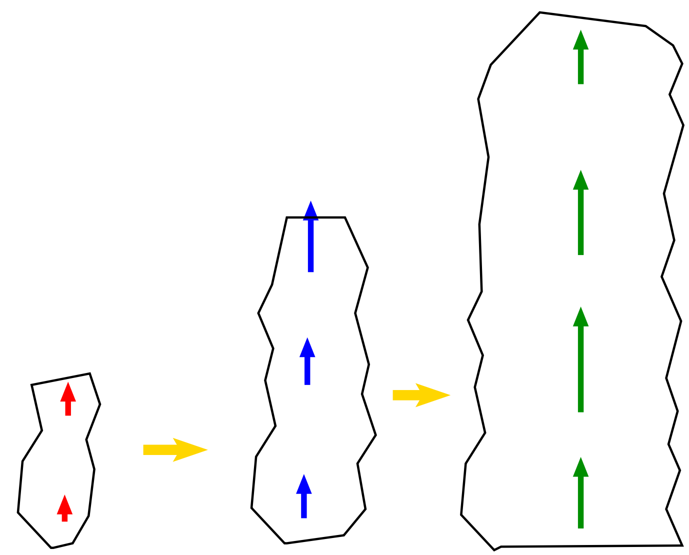

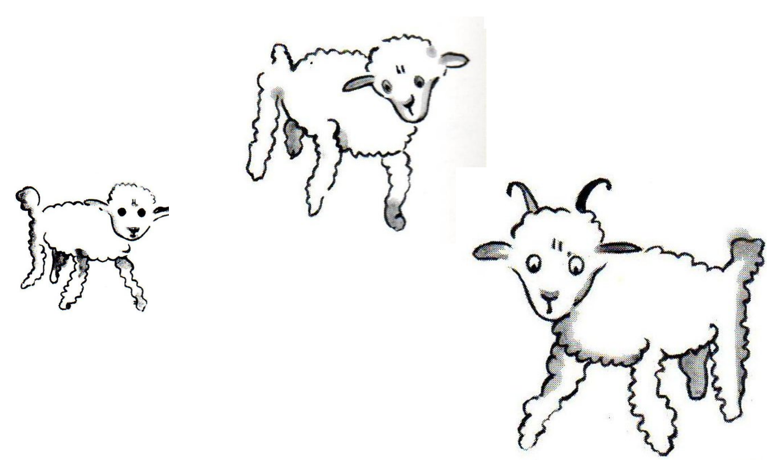

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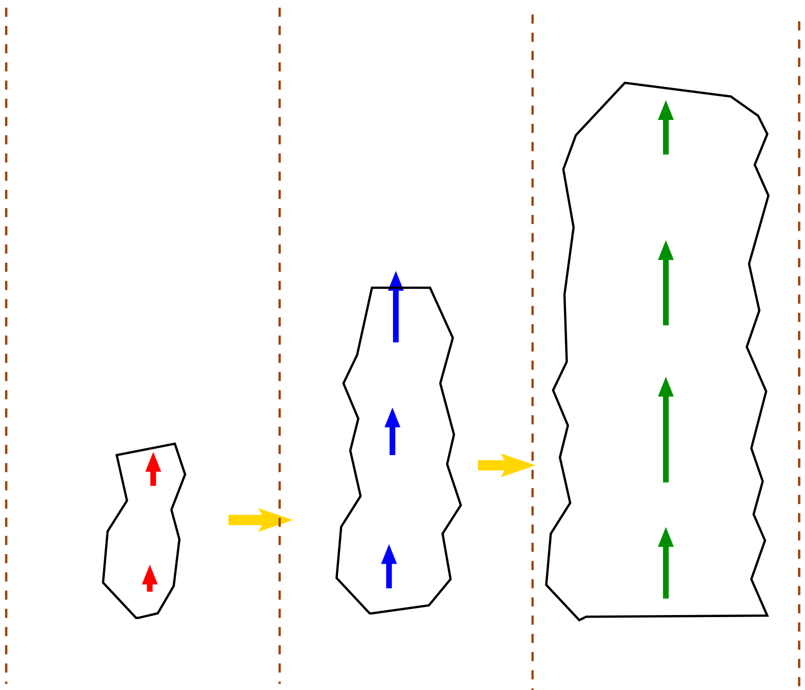
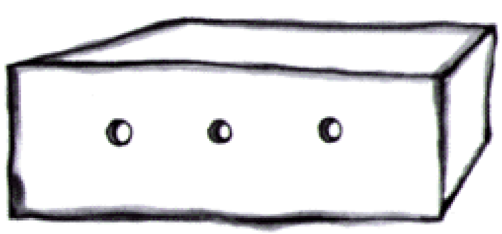
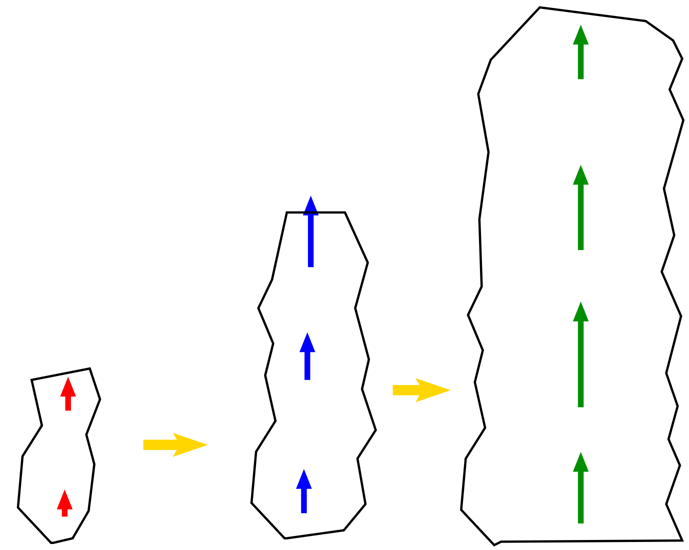
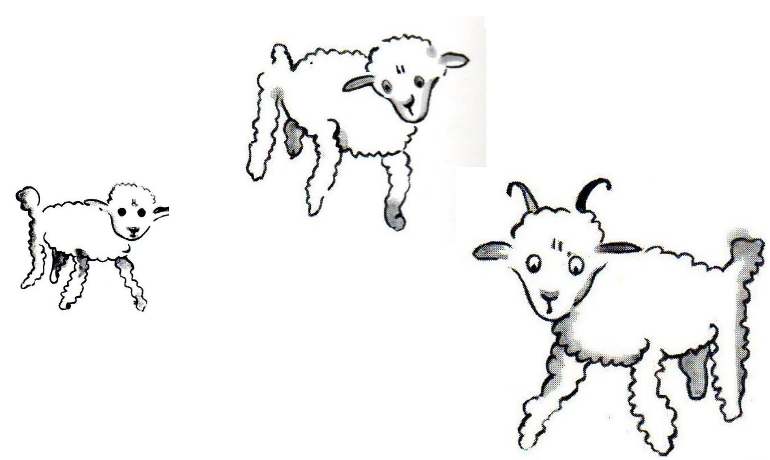
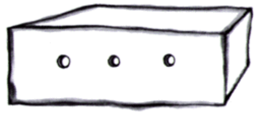




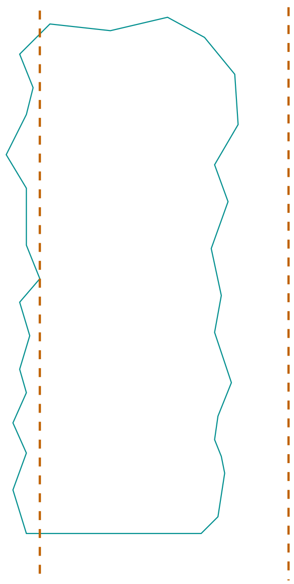






Box story - handle with care

Resolved is blind – SG acts locally after looking globally.



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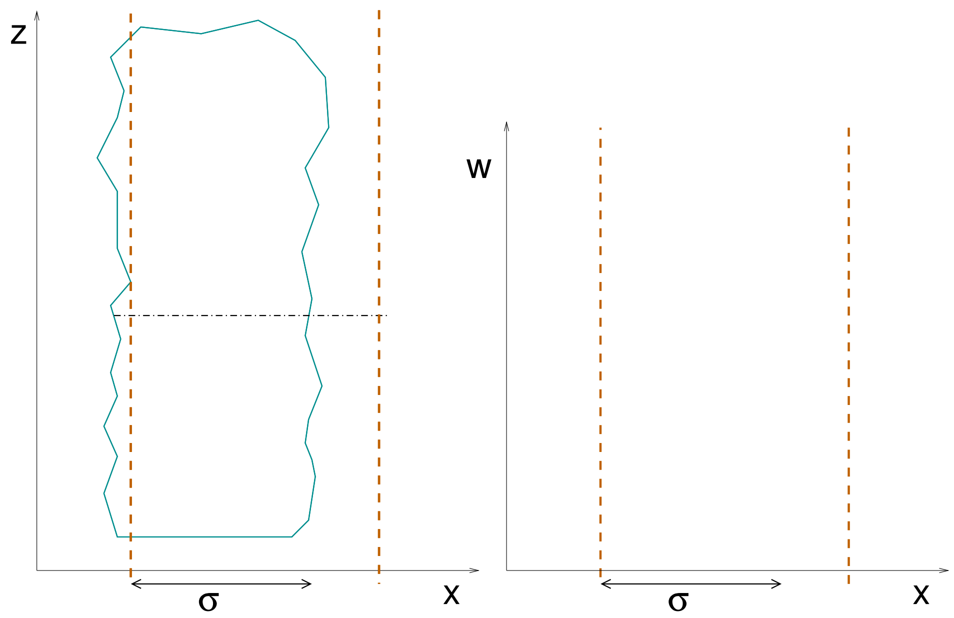

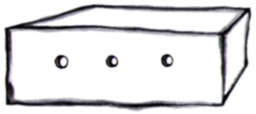

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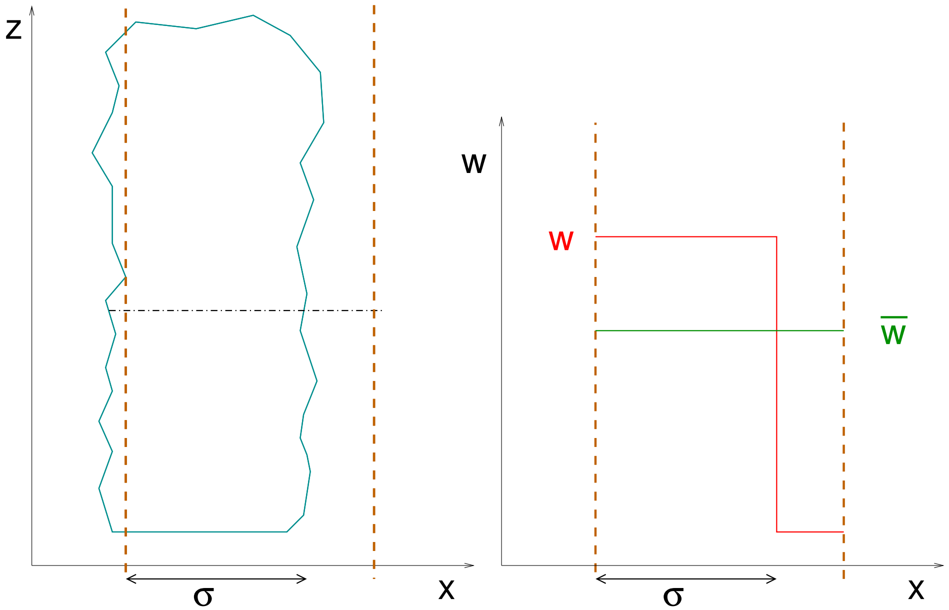

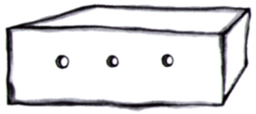

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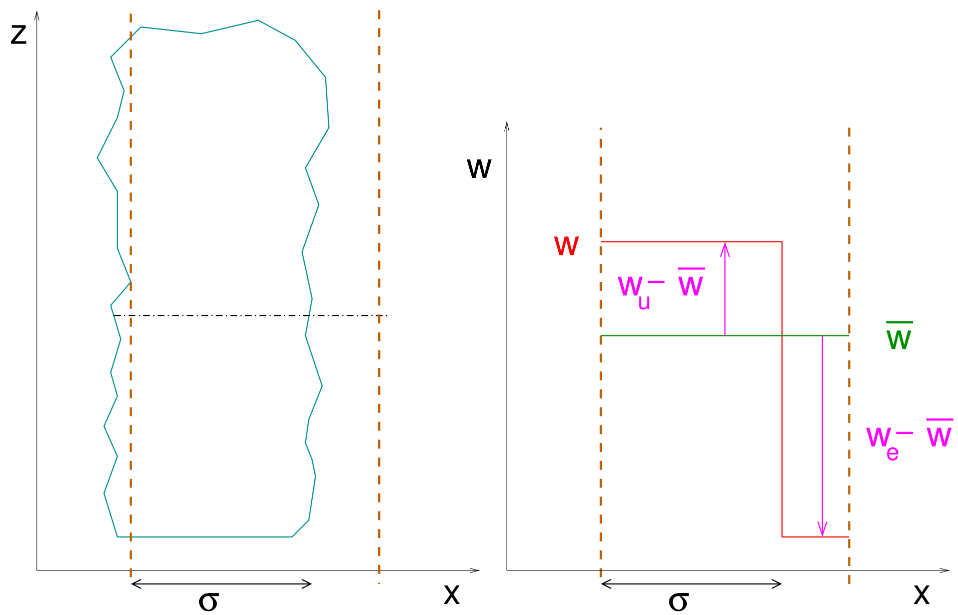
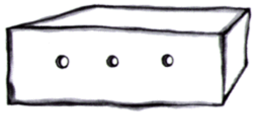


$$\sigma_u(\omega_u - \overline{\omega}) + (1-\sigma_u)(\omega_e - \overline{\omega}) = 0$$

subgrid relative velocity : $\omega_u^\diamond = \omega_u - \overline{\omega}$

RMI



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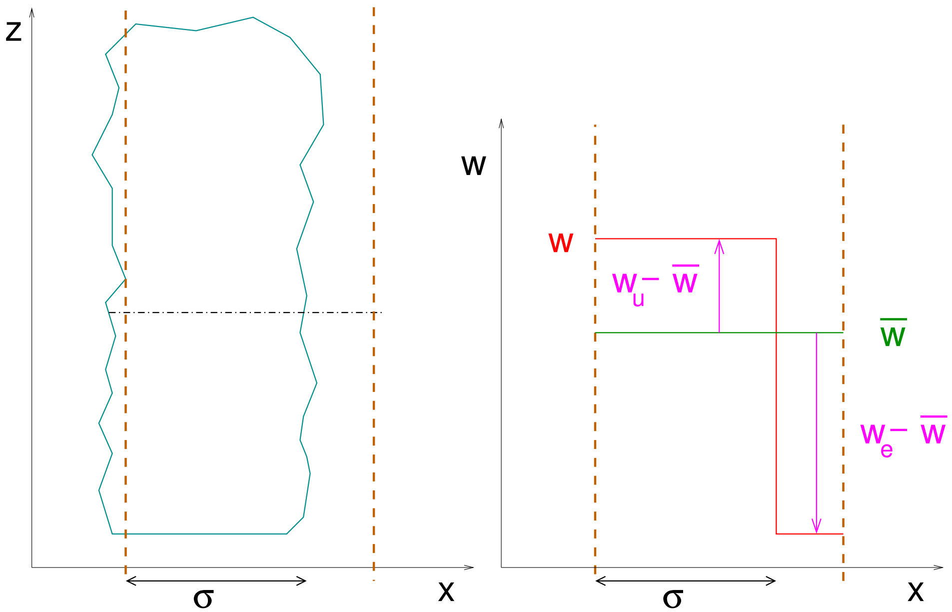

Virtual Unresolved Cloud :
- condenses with $\sigma_u(\omega_u - \overline{\omega})$
- Transports with $\sigma_u(\omega_u - \omega_e)$
- Entrains with $\sigma_u(\omega_u - \omega_e)$
- Rises with ω_u

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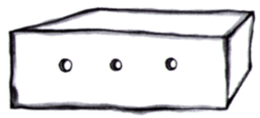

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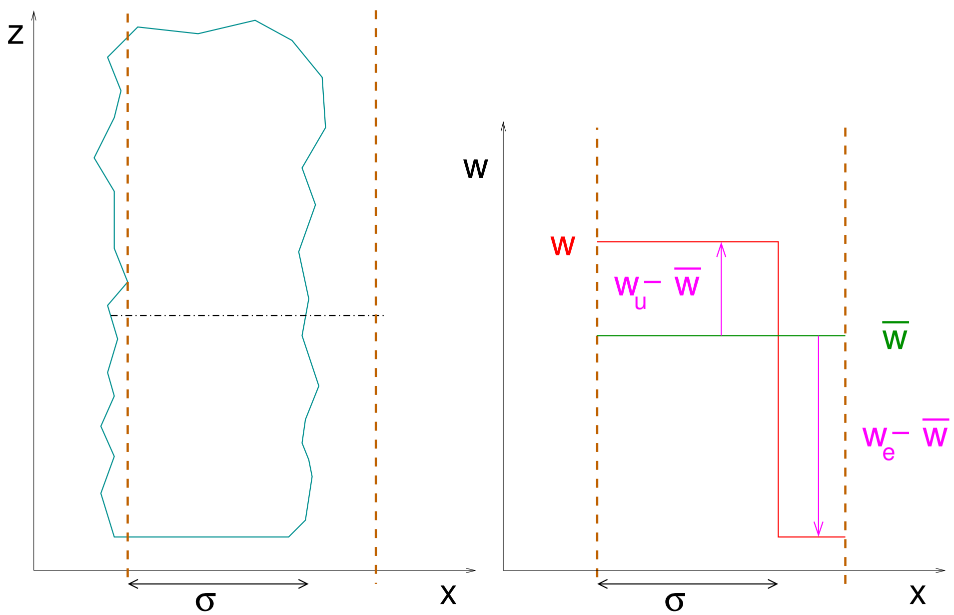

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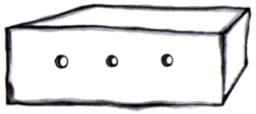
Newton law formulation
no more 'apparent mass coefficient'

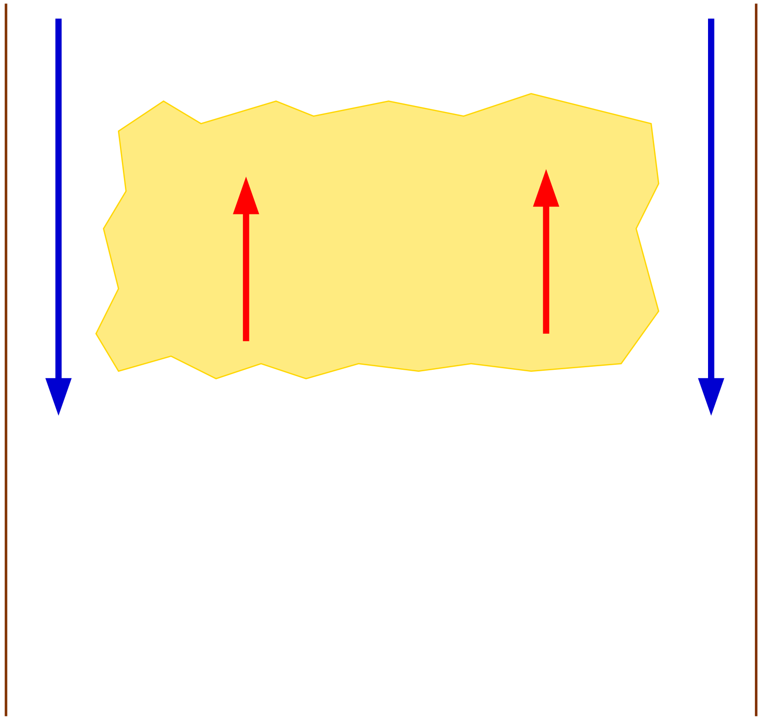
$$\frac{d\big[\sigma_u(\omega_u - \overline{\omega})\big]}{dt} - \frac{d\big[(1-\sigma_u)(\omega_e - \overline{\omega})\big]}{dt} = (\mathbf{F_b} + \text{drag})$$

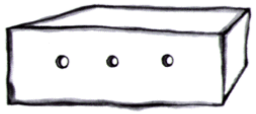
$$2\frac{d\big[\sigma_u \omega_u^{\diamond}\big]}{dt} = (\mathbf{F_b} + \text{drag})$$

$$\boxed{\omega_u^{\diamond} = \omega_u - \overline{\omega}}$$



Box story - The voice of the elders





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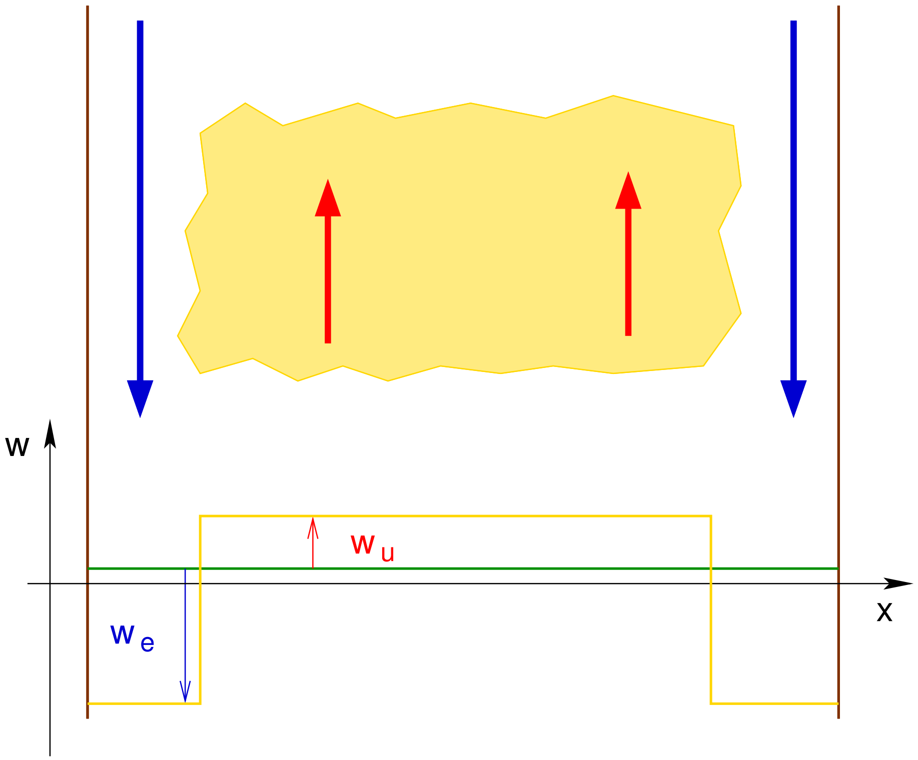

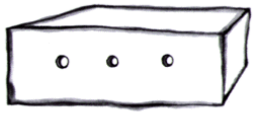

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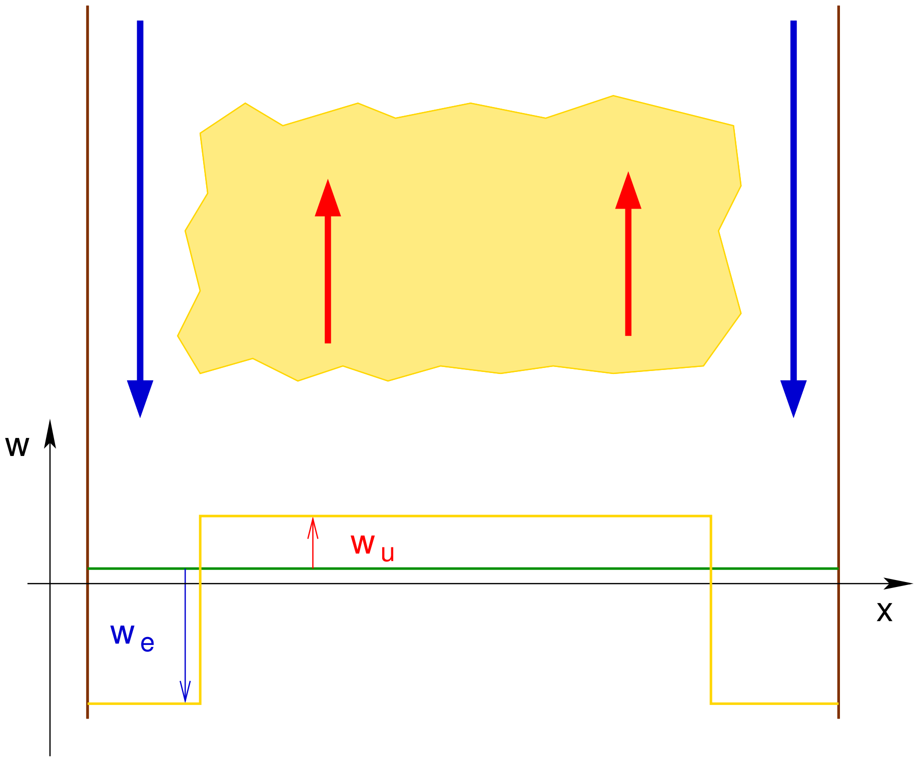
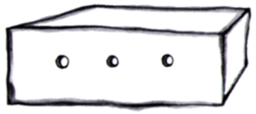


Bjerknes (1938), Asai and Kasahara (1967)

$$\frac{\partial T_u}{\partial t} \approx -w_u \frac{g}{c_p}\frac{\partial h}{\partial \phi} \qquad \leq 0$$

$$\frac{\partial T_e}{\partial t} \approx -w_e \frac{g}{c_p}\frac{\partial s}{\partial \phi} \qquad \geq 0$$



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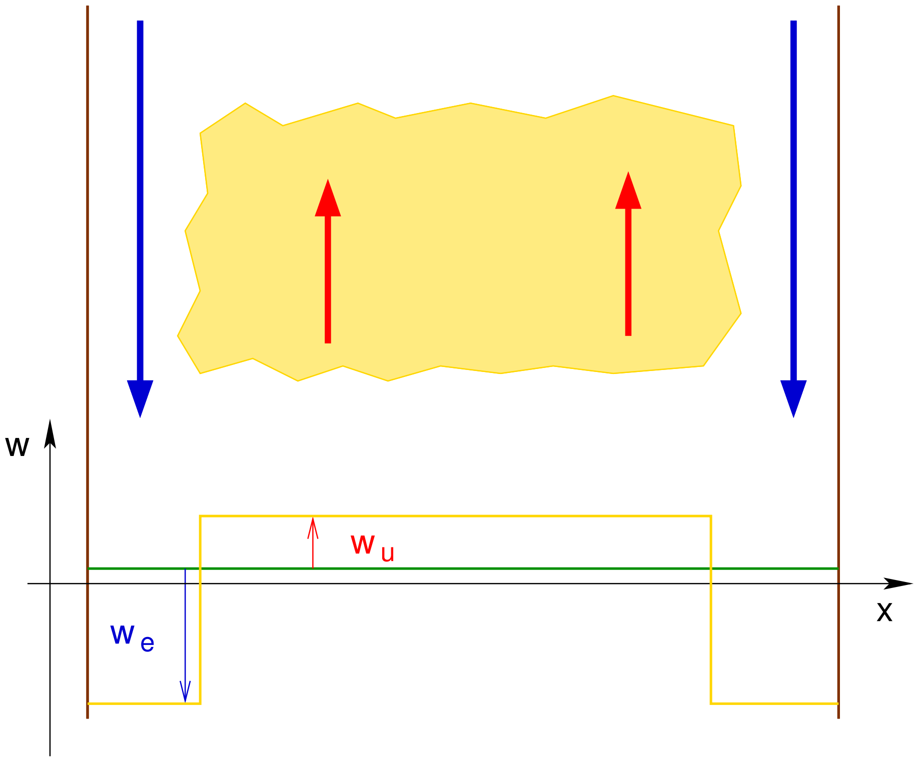

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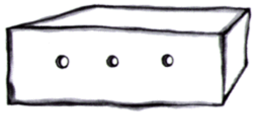
$$T_{vu} - \overline{T_v}^{+} \approx (T_{vu} - \overline{T_v})\left[1 - \sigma_u \underbrace{(1 - \frac{\triangle s}{\triangle h})}_{b \geq 1}\right]$$

Bjeknes buoyancy-reduction coefficient $b \geq 1$



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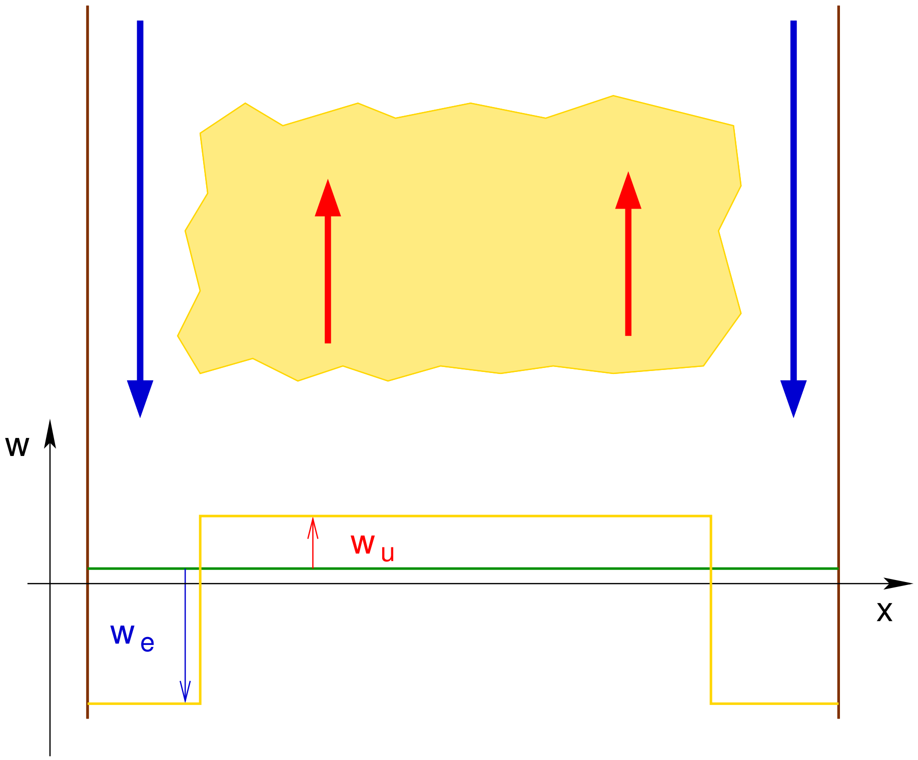

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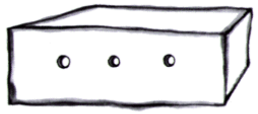
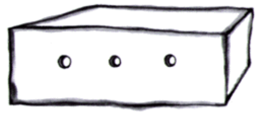


BBR in practice

Extrapolating local gradients upwards and downwards is inadequate.

$$\mathbf{b} = 1 - \min(0, \frac{\dfrac{h_e^l - h_e^b}{p^b - p^l} \quad (\leq 0)}{\dfrac{s_e^t - s_e^l}{p^l - p^t} \quad (\geq 0)})$$



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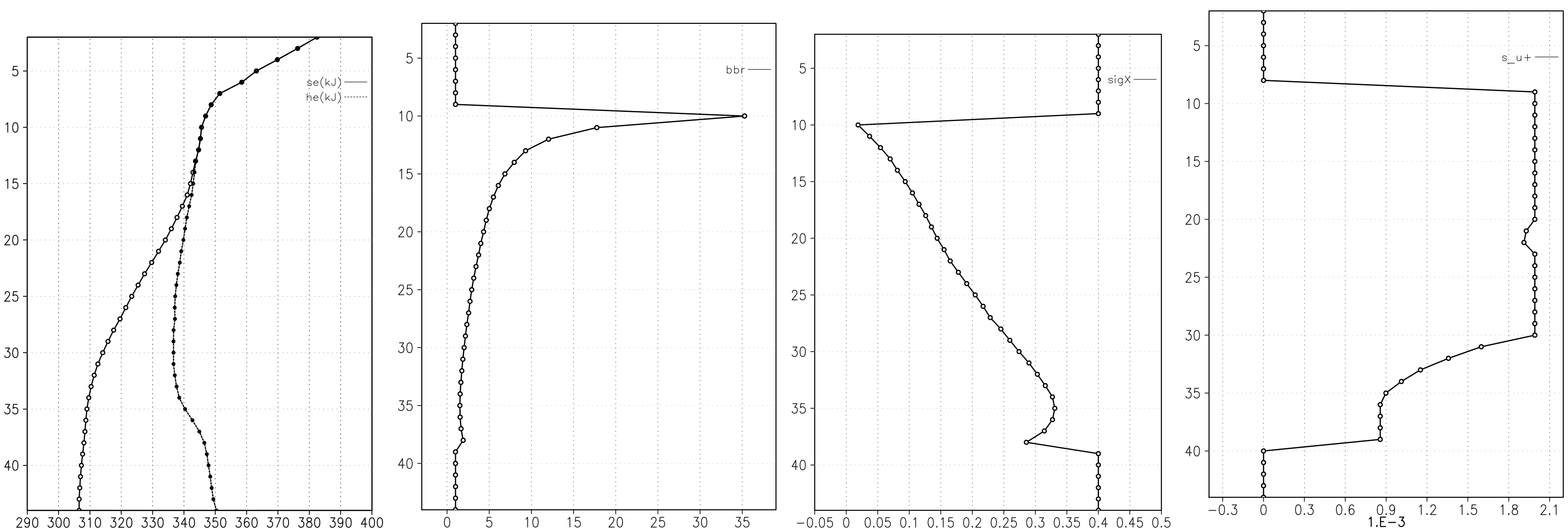

Buoyancy, Drag and entrainment

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Buoyancy expresses forces between updraught and environment

$$m\frac{dw}{dt} = \mathbf{F_u} = -\mathbf{F_e} \qquad \Longrightarrow \qquad \mathbf{F_b} = -\sigma_u g^2 \frac{p}{R_a} \frac{T_{vu} - \overline{Tv}}{\overline{T_v} T_{vu}} (1 - b\sigma_u)$$

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Entrainment concerns processes at the cloud-environment interface

$$\frac{\triangle M_u}{M_u} = \lambda_u \triangle \phi = \frac{E_u \triangle p}{M_u}$$

$$M_u = \sigma_u(\omega_u - \omega_e) = \sigma_u \omega_u^* = \frac{\sigma_u}{1 - \sigma_u} \omega_u^\diamond$$

$$m\frac{dw}{dt} = (w_u - w_e)\triangle M_u$$

$$\text{drag} = \sigma_u \frac{R_a T_{vu}}{p} \left(\lambda_u + \frac{\mathcal{K}_{du}}{g}\right) \frac{{\omega_u^\diamond}^2}{(1 - \sigma_u)^2}$$

Total derivative includes advection

$$\frac{d\left[\sigma_u\omega_u^\diamond\right]}{dt} = \frac{\partial\left[\sigma_u\omega_u^\diamond\right]}{\partial t}\bigg|_{\mathrm{phys}} + \frac{\partial\left[\sigma_u\omega_u^\diamond\right]}{\partial t}\bigg|_{\mathrm{dyn}} + \mathbf{V}\cdot\nabla_\eta\left[\sigma_u\omega_u^\diamond\right] + \dot{\eta}_u\frac{\partial p}{\partial\eta}\frac{\partial\left[\sigma_u\omega_u\right]}{\partial p} - \dot{\overline{\eta}}\frac{\partial p}{\partial\eta}\frac{\partial\left[\sigma_u\overline{\omega}\right]}{\partial p}$$

RMI

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combined with resolved advection of $\omega_u^\diamond$ and σ_u

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RMI

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Resolved advection of σ_u and $\omega_u^\diamond$ is necessary
– to get the cloud moving with the wind
– to eliminate the horizontal advective term from the subgrid tendency.
– Vertical shear mixes different columns.
– S.L. advection means interpolating origin points.

RMI

Complete motion equation

Complete motion equation

$$\frac{\partial \omega_u^\diamond}{\partial t}\Big|_{\rm phys} = -\frac{1}{2} g^2 \frac{p(T_{vu} - \overline{T_v})}{R_a T_{vu} \overline{T_v}} (1 - \mathbf{b}\sigma_u) + \frac{1}{2}\frac{R_a T_{vu}}{p}\Big(\lambda_u + \frac{\mathcal{K}_{du}}{g}\Big)\frac{{\omega_u^\diamond}^2}{(1-\sigma_u)^2}$$

$$- \omega_u^\diamond \Big(\underline{\underline{\frac{\partial \omega_u^\diamond}{\partial p} + \frac{\partial \overline{\omega}}{\partial p} + (\omega_u^\diamond + \overline{\omega}) \frac{\partial \ln \sigma_u}{\partial p}}} + \frac{\partial \ln \sigma_u}{\partial t} \Big)$$

RMI

Complete motion equation

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$$

$$
- \, \omega_u^{\diamond} \Big(\underline{\underline{\frac{\partial \omega_u^{\diamond}}{\partial p} + \frac{\partial \overline{\varpi}}{\partial p} + (\omega_u^{\diamond} + \overline{\varpi}) \frac{\partial \ln \sigma_u}{\partial p}}} + \frac{\partial \ln \sigma_u}{\partial t} \Big)
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Auto-advection terms are critical at rising top.

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Auto-advection terms are critical at rising top.

For this we need first to gather some more tools.

Membership/classification - how an ascent is built

$(\overline{T_w}^l, \overline{q_w}^l)$ elevated to next level above : (T_u^{l-1}, q_u^{l-1})

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$$T_u^{l-1} > T_{we}^{l-1} \Longrightarrow \delta_{\mathrm{asc}}^{l-1} = 1 \text{ else back to blue point } \delta_{\mathrm{asc}}^{l-1} = 0$$

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RMI

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RMI

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– Base level :
  – $\delta_{\text{bas}}^l = \delta_{\text{asc}}^l (1 - \delta_{\text{asc}}^{l+1})$

  – $\delta_{\text{bas}}^l = \delta_{\text{bu}}^l (1 - \delta_{\text{bu}}^{l+1})$
    better for sub-base above a local CIN barrier.

RMI

Top evolution : activity index

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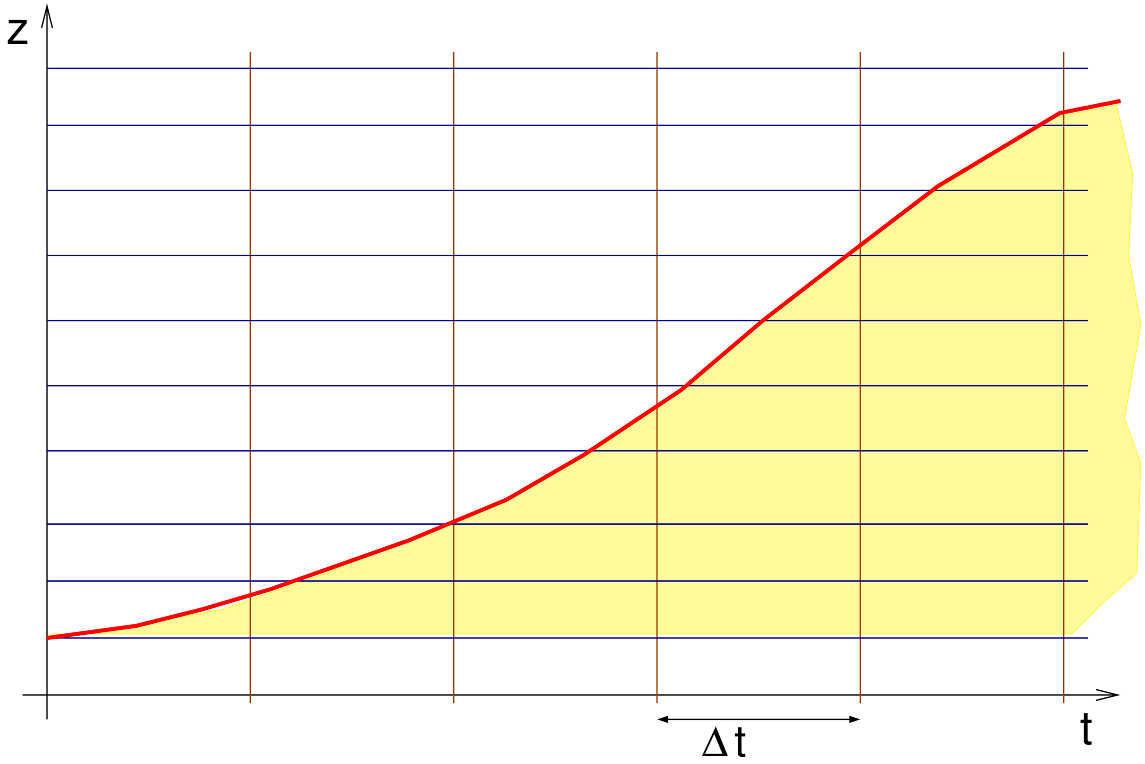

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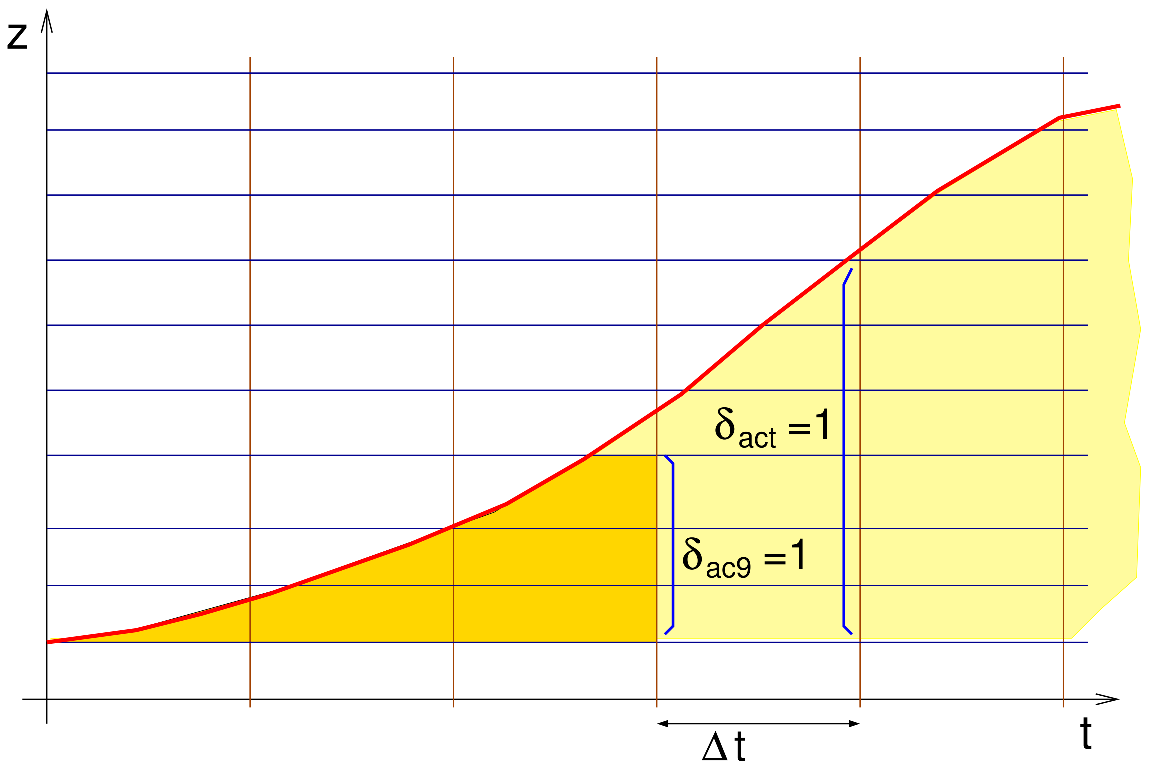
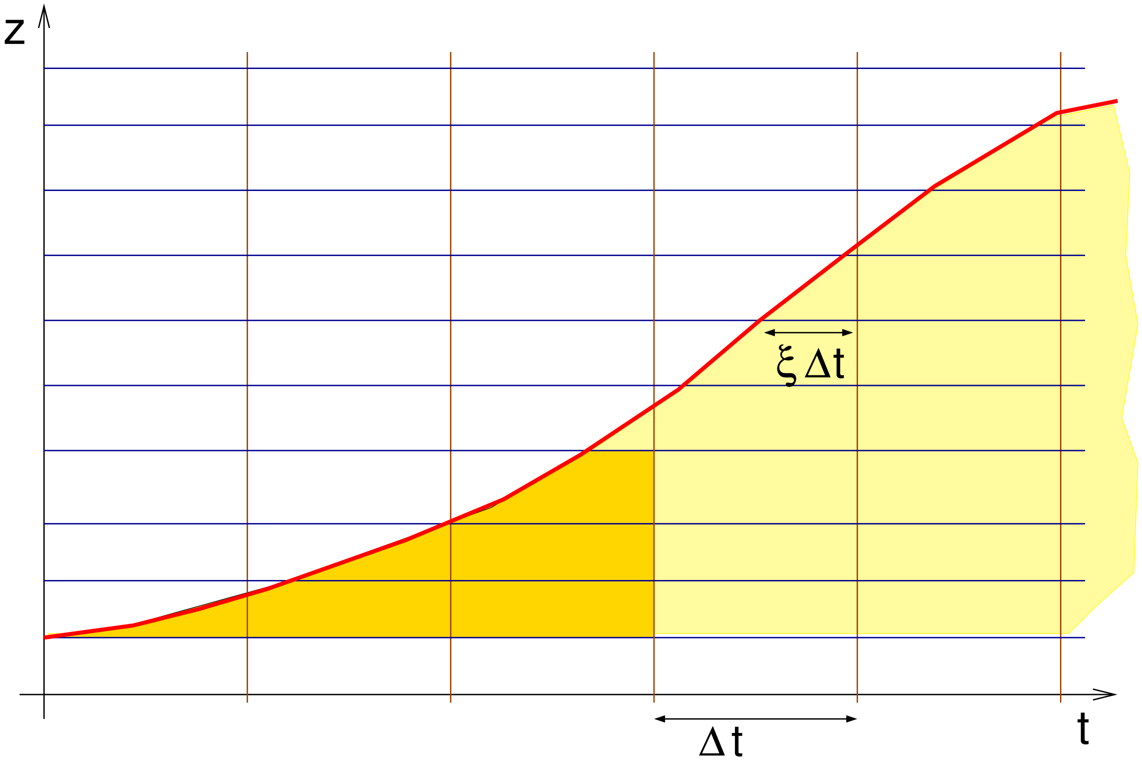


$\delta_{\mathrm{act}} = 1$ at levels reached by the ascent originating at the base

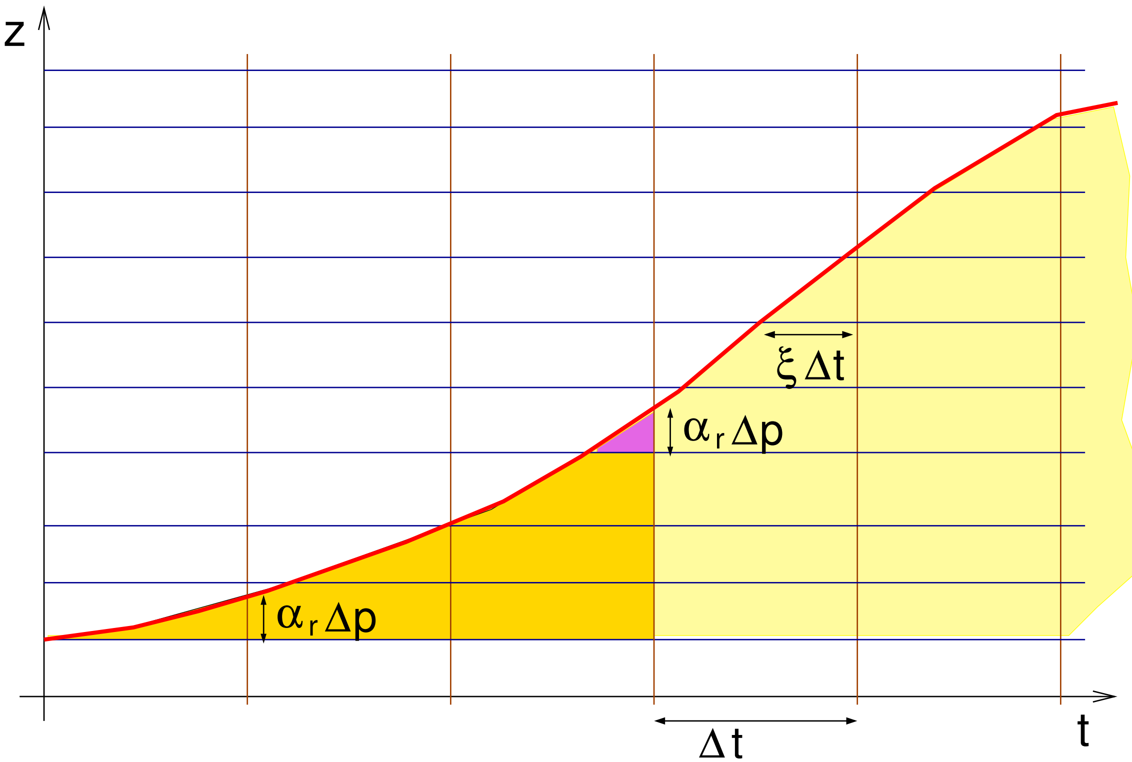
δ_{ac9} retrieved from profile of ω_u^- or σ_u^-

Top evolution : activity index



Buoyancy accelerates the fluid during $\xi \triangle t$

Top evolution : activity index

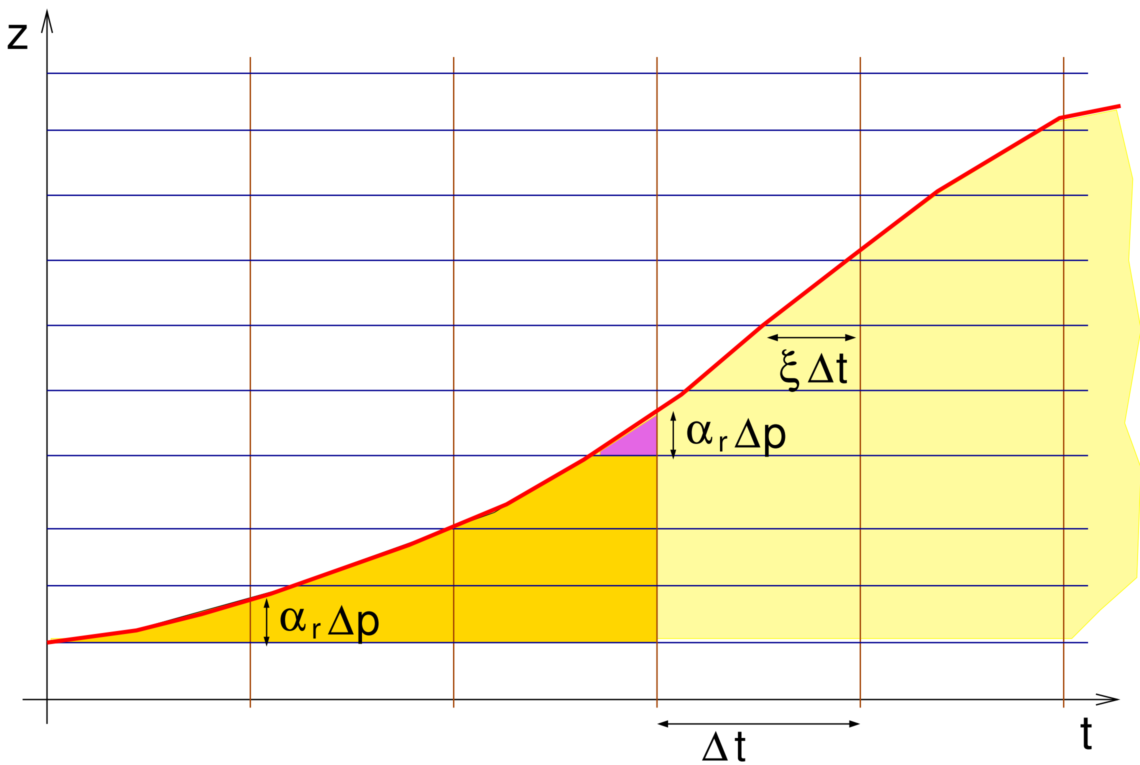


$\delta_{\rm ac9}$, $\delta_{\rm act}$ record the discrete evolution of cloud vertical extension

ξ diagnosed for estimating time-averaged and final states

α_r records fractional path above upper last active level

Top evolution : activity index



δ_{ac9}, δ_{act} record the discrete evolution of cloud vertical extension

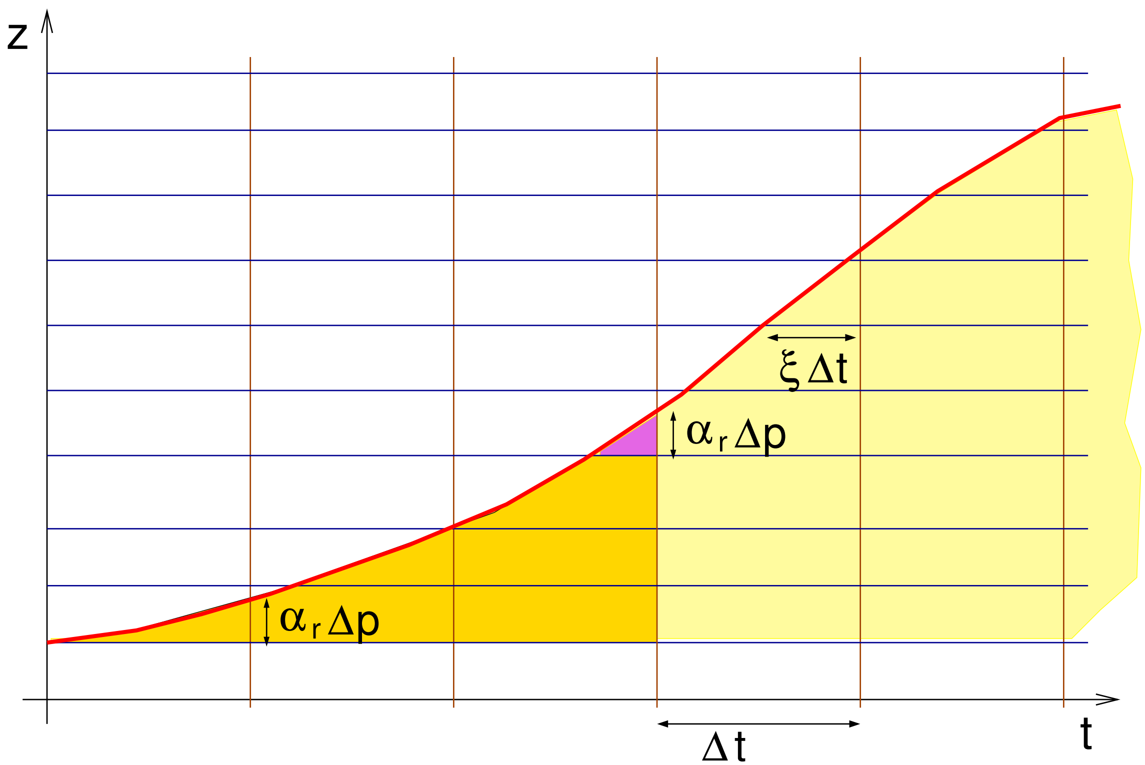
ξ diagnosed for estimating time-averaged and final states

α_r records fractional path above upper last active level

- α_r is necessary for *initiating* an updraught with $|\omega_u|$ small ;
- is necessary to compute ξ ;
- is associated to a single cloud top :
  top level detected in advected variables (ω_u, σ_u), and can move its position following resolved advection.
- α_r cannot be interpolated between different columns.

RMI

Top evolution : activity index



δ_{ac9}, δ_{act} record the discrete evolution of cloud vertical extension

ξ diagnosed for estimating time-averaged and final states

α_r records fractional path above upper last active level

Idea : use a single α_r for the column,
memorized in a local pseudo-historical variable :
– not advected, no interpolation ;
– corresponding to the 'main' updraught segment.

Complete motion equation (bis)

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$$\frac{\partial \omega_u^\diamond}{\partial t}\Big|_{\text{phys}} = -\xi^l \frac{1}{2} g^2 \frac{p(T_{vu} - \overline{T_v})}{R_a T_{vu} \overline{T_v}} (1 - \mathbf{b}\sigma_u) + \xi^l \frac{1}{2} \frac{R_a T_{vu}}{p} \left(\lambda_u + \frac{\mathcal{K}_{du}}{g}\right) \frac{\omega_u^{\diamond 2}}{(1-\sigma_u)^2}$$

$$- \delta_{\text{ac9}}^l \omega_u^{\diamond l} \left(\frac{\partial \omega_u^\diamond}{\partial p} + \frac{\partial \overline{\omega}}{\partial p} + (\omega_u^\diamond + \overline{\omega}) \frac{\partial \ln \sigma_u}{\partial p} + \frac{\partial \ln \sigma_u}{\partial t} \right)$$

$$- (1 - \delta_{\text{ac9}}^l) \omega_u^{\diamond l+1} \left\{ \xi^l \left(\frac{\partial \overline{\omega}}{\partial p} + \frac{\partial \ln \sigma_u}{\partial t} \right) + \underline{(\xi^{l+1} - \xi^l) \frac{(\omega_u^\diamond + \overline{\omega})^{l+1}}{p^{l+1} - p^l}} \right\}$$

RMI

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$$- (1 - \delta_{\text{ac9}}^l)\omega_u^{\diamond l+1}\left\{\xi^l\left(\frac{\partial \overline{\omega}}{\partial p} + \frac{\partial \ln \sigma_u}{\partial t}\right) + \underline{(\xi^{l+1} - \xi^l)\frac{(\omega_u^\diamond + \overline{\omega})^{l+1}}{p^{l+1} - p^l}}\right\}$$

At newly active levels $0 < \xi^l < 1$, $\delta_{\text{ac9}}^l = 0$, and

$$(\xi^{l+1} - \xi^l) = \frac{(p^{l+1} - p^l)}{\omega_u^{l+1} \triangle t} \Longrightarrow \omega_u^{\diamond l^+} \approx \omega_u^{\diamond l+1} + \ldots$$

RMI

Steady-state motion equation

$$\left.\frac{\partial \omega_u^\diamond}{\partial t}\right|_{\text{phys}} = 0 = -F(1-\mathbf{b}\sigma_u) + K\frac{\omega_u^{\diamond 2}}{(1-\sigma_u)^2} - \omega_u^\diamond \beta$$

– Without auto-advection : $\beta \sim 0 \Longrightarrow \boxed{\omega_u^{\diamond\|} = -(1-\sigma_u)\sqrt{1-\mathbf{b}\sigma_u}\ \sqrt{\frac{F}{K}}}$

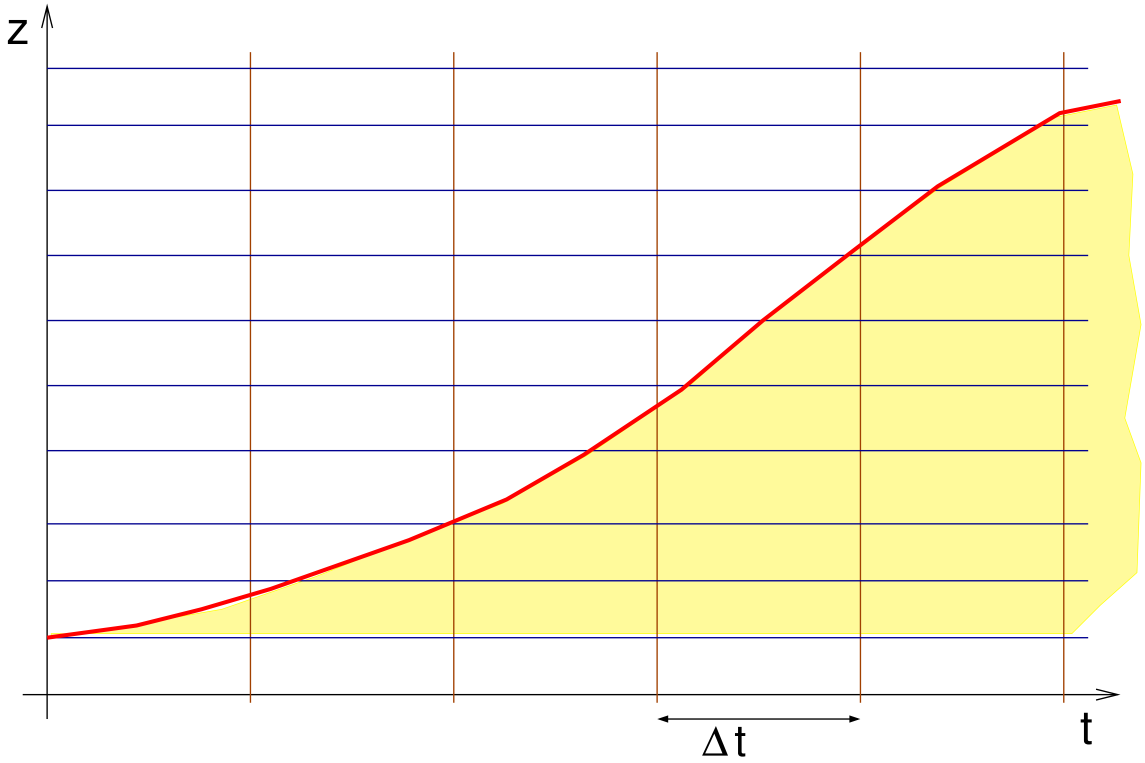
– If we have a guess for $\beta = \left(\frac{\partial \omega_u}{\partial p} + \omega_u \frac{\partial \ln \sigma_u}{\partial p} + \frac{\partial \ln \sigma_u}{\partial t}\right) \approx \frac{\partial \omega_u}{\partial p}$:

$$\omega_u^{\diamond\|\,2} - \frac{F}{K}(1-\mathbf{b}\sigma_u)(1-\sigma_u)^2 - \frac{\beta}{K}\omega_u^{\diamond\|}(1-\sigma_u)^2 = 0$$

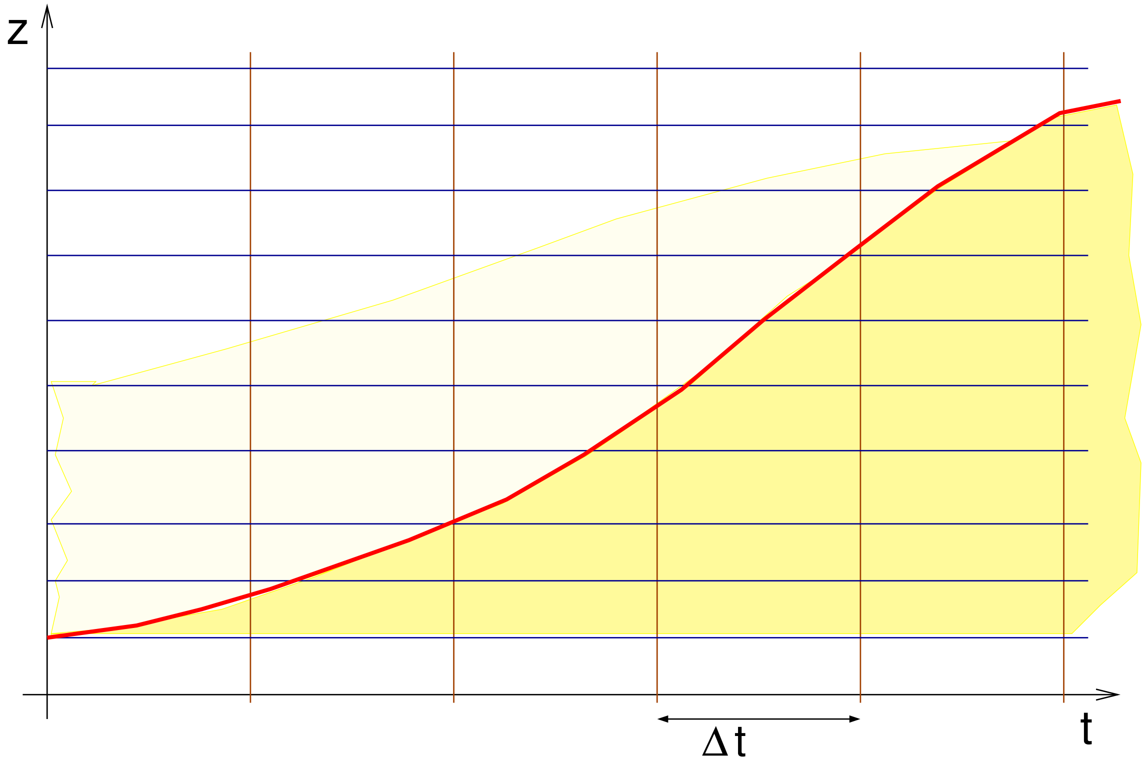
$$\omega_u^{\diamond\|} \sim \frac{\beta}{2K}(1-\sigma_u)^2 - (1-\sigma_u)\sqrt{\left(\frac{\beta}{2K}\right)^2(1-\sigma_u)^2 + \frac{F}{K}(1-\mathbf{b}\sigma_u)}$$

Base and secondary ascents vs triggering

Base and secondary ascents vs triggering



Base and secondary ascents vs triggering



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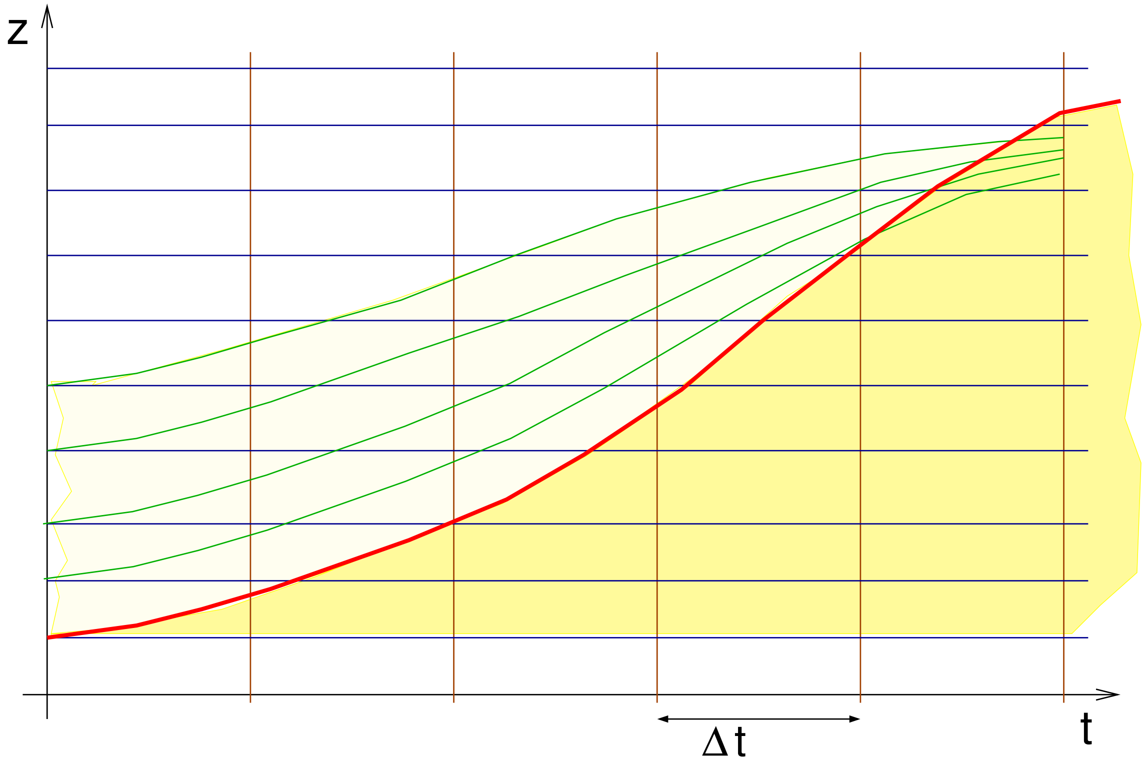

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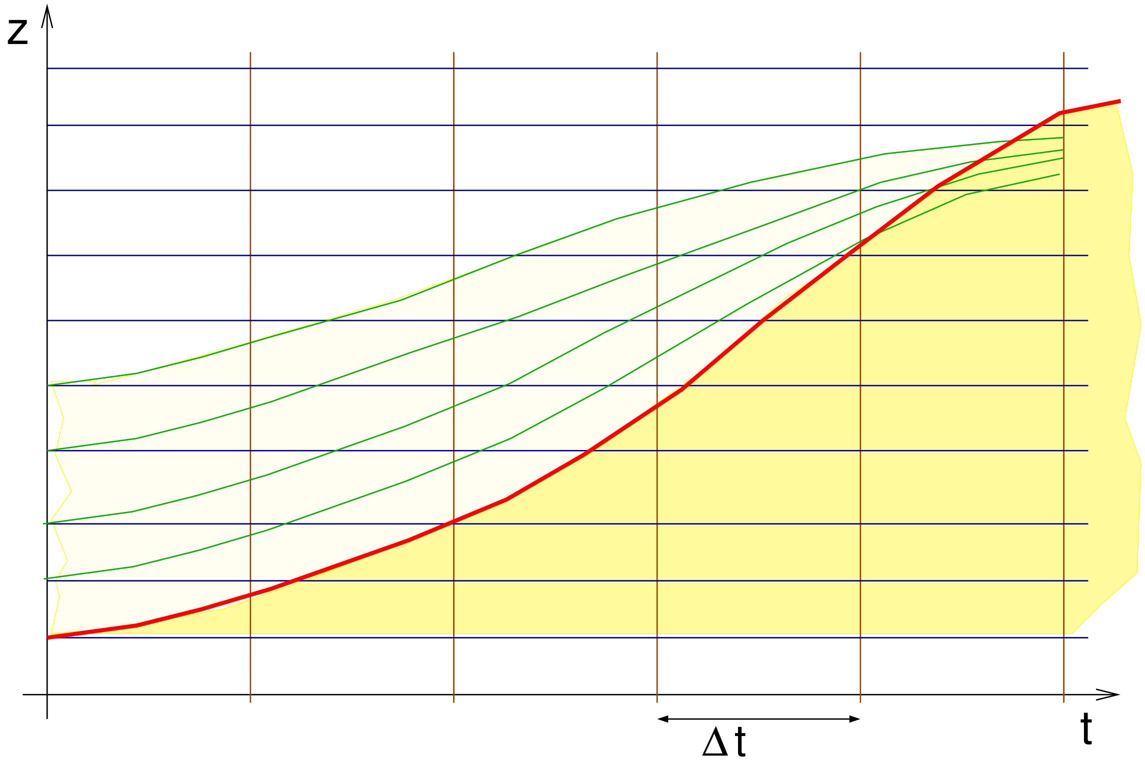
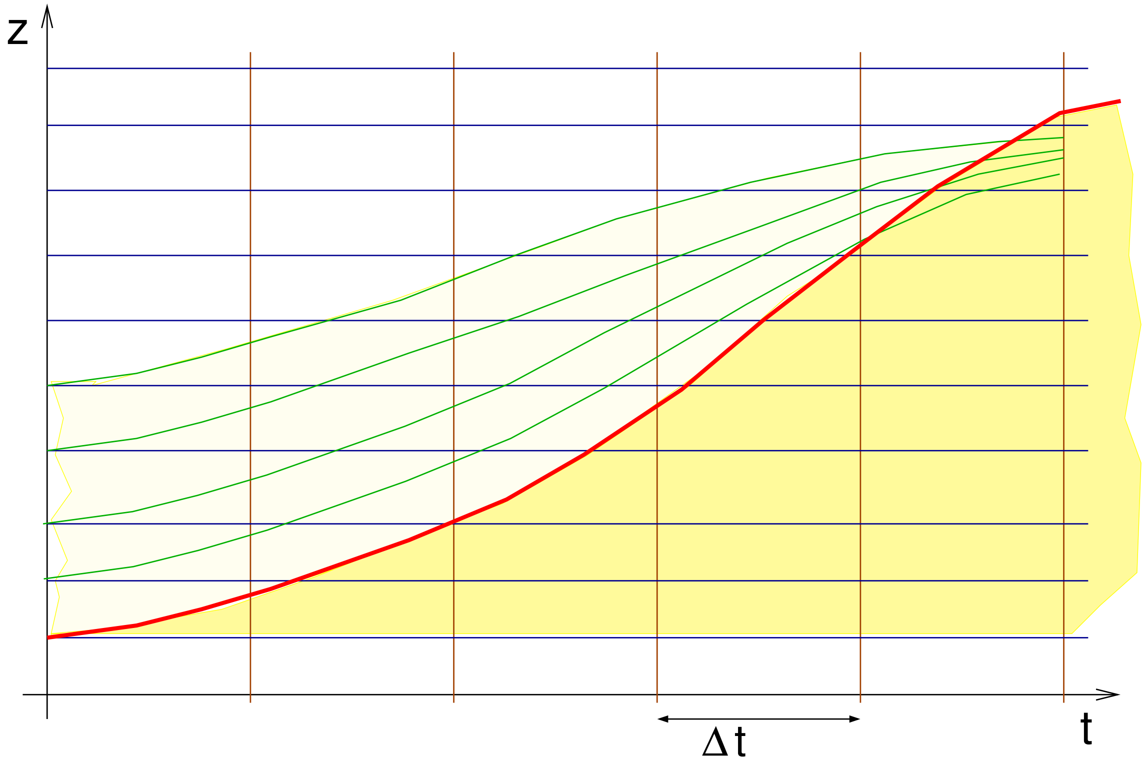


In principle ascents could start from various level at the same time ;

Base and secondary ascents vs triggering



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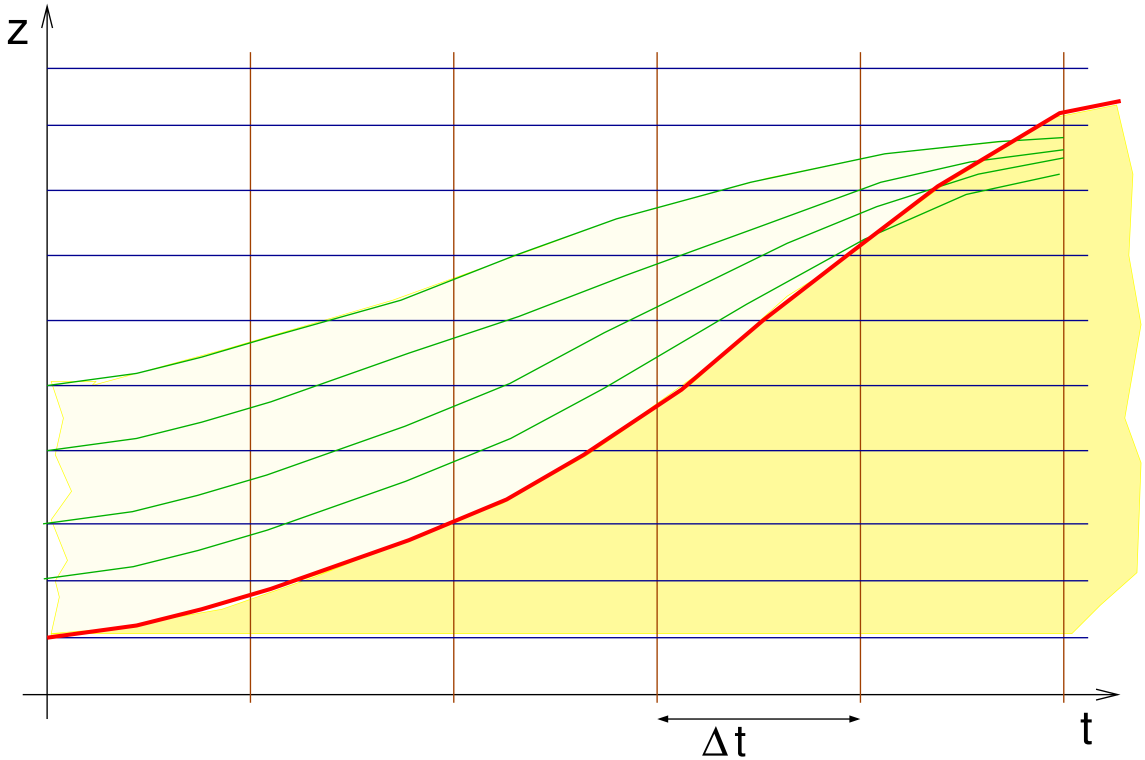
But

- they should be catched up by the ascent originating from the base ;
- actual updraught triggering rather starts from the Boundary layer.

Base and secondary ascents vs triggering



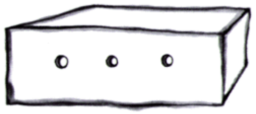
In principle ascents could start from various level at the same time ;

But

- they should be catched up by the ascent originating from the base ;
- actual updraught triggering rather starts from the Boundary layer.

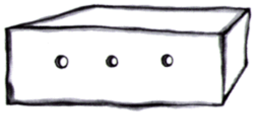
If this covers some reality (?)
its treatment appears feasible only in still atmosphere
(no sheared advection / no mixing).

Closure : a steady-state diagnostic



Current closure relations express an equilibrium.
Larger-scale ‘forcing’ $\longrightarrow$ subgrid scheme response

Closure : a steady-state diagnostic

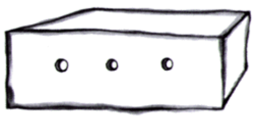


Current closure relations express an equilibrium.
Larger-scale 'forcing' $\longrightarrow$ subgrid scheme response

q_v convergence $\rightarrow$

Diagnostic closure
scaling of M_u in steady state
latent heat release by condensation
vertical re-organization :
transport, condensation, precipitation, detrainment
'somewhere in the grid-column'

Closure : a steady-state diagnostic

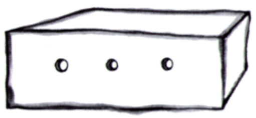


Current closure relations express an equilibrium.
Larger-scale 'forcing' $\longrightarrow$ subgrid scheme response

q_v convergence $\rightarrow$

Prognostic closure
scaling of M_u in steady state
way to this :
latent heat storage by increasing $\sigma_u(h_u - h_e)$
$+$ latent heat release by condensation
entrainment and condensation
'somewhere in the grid-column'
$\neq$ level the vapour entered the column

Closure : a steady-state diagnostic



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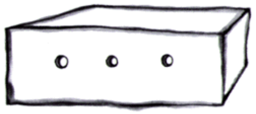
High resolution

$*$ resolved scheme $\rightarrow \begin{cases} \text{excess of } q_v > q_{\mathrm{sat}} \\ \text{decrease of } q_{\mathrm{sat}}\ (\overline{\omega}\uparrow,\ T\searrow) \\ \textit{not limited to MoCon} \end{cases}$

$*$ Subgrid scheme $\rightarrow \begin{cases} \text{condensation} \\ \text{storage in } \sigma_u \text{ extension} \end{cases}$

short $\triangle t$, small $\triangle x$
$\Longrightarrow$ tendency of q_v, q_t over the column may be $\neq 0$.

Closure : a steady-state diagnostic



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Rising cloud :

* MoCon feeds resolved *and* subgrid schemes, $\neq$ at $\neq$ levels ;
* Resolved condensation not limited to new moisture arrival.

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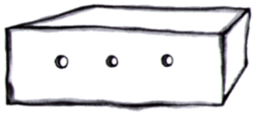

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Writing a MOCON steady-state closure

$$\int_{p_t}^{p_b} \sigma_u(\omega_u^{\diamond\|} + \overline{\omega}) \frac{\delta q_{ca}}{g} = - \int_{p_t}^{p_b} \mathsf{CVGQ} \frac{dp}{g}$$

Normalized mass flux :

$$\mu = \frac{M_u}{M_B} = \frac{\sigma_u \omega_u^{\diamond}}{\sigma_b \omega_b^{\diamond}} \qquad \Longrightarrow \qquad M_B = - \frac{\int_{p_t}^{p_b} \mathsf{CVGQ} \frac{dp}{g} + \int_{p_t}^{p_b} \sigma_u \overline{\omega} \frac{\delta q_{ca}}{g}}{\int_{p_t}^{p_b} \mu \delta q_{\mathrm{ca}}}$$

Closure yields M_B, then get steady-state mesh fraction by solving

$$\sigma_u \omega_u^{\diamond\|} = -\sigma_u (1 - \sigma_u) \sqrt{1 - \mathbf{b} \sigma_u} \sqrt{\frac{F}{K}} = \mu M_B$$

RMI

CAPE diagnostic closure

Nordeng's (1994) CAPE closure

$$\left.\frac{\partial \mathsf{CAPE}}{\partial t}\right|_{ud} = -\frac{\mathsf{CAPE}}{\tau}$$

$$\left.\frac{\partial \mathsf{CAPE}}{\partial t}\right|_{ud} \approx -\int g \left.\frac{\partial \overline{\theta}}{\partial t}\right|_{ud} \frac{dp}{\rho g} \approx -\int M_u \frac{\partial \overline{\theta}}{\partial p} \frac{dp}{g}$$

can only be estimated assuming a steady-state updraught :
$\frac{\partial \theta_{uv}}{\partial t} \sim 0$, $\overline{\theta} \sim \overline{\theta_v}$, M_u up to the equilibrium level and does no longer vary during τ.

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MTCS : Transport and Condensation

$$-\left.\frac{\partial \overline{T}}{\partial t}\right|_{\mathrm{ud}} = \frac{1}{c_p}\left\{\frac{\triangle\left(\frac{\sigma_u}{1-\sigma_u}\omega_u^{\diamond}(s_u - \overline{s})\right)}{\triangle p} + L\frac{\sigma_u(\omega_u^{\diamond} + \overline{\omega})\delta q_{ca}}{\triangle p}\right\}$$

RMI

CAPE diagnostic closure

$$T_v \approx T(1+\nu q_v), \qquad \mu = \frac{M_u}{M_B} = \frac{\sigma_u \omega_u^\diamond}{\sigma_b \omega_b^\diamond}$$

$$M_B \sum \Big\{ \frac{1}{p} \triangle \Big[\frac{\mu}{1-\sigma_u} \Big(\frac{s_u - \overline{s}}{c_p} + \nu T(q_u - \overline{q}) \Big) \Big] \Big\} + M_B \sum \Big\{ \frac{\mu \delta q_{ca}}{p} \Big[\frac{L}{c_p} - \nu T \Big] \Big\}$$

$$= \frac{1}{\tau} \sum (T_{vu} - \overline{T_v}) \frac{\triangle p}{p} + \sum \frac{\sigma_u \overline{\omega} \delta q_{ca}}{p} \Big[\frac{L}{c_p} - \nu T \Big]$$

Normalized mass flux

Entrainment/Detrainment associated to $M_u^* = \sigma_u(\omega_u - \omega_e)$.

$$\frac{\partial \ln M_u^*}{\partial p} = (\lambda_u - \kappa_u)\frac{\triangle\phi}{\triangle p} \qquad \Longrightarrow \qquad \mu^{*l} = \mu^{*l+1} \exp\big((\lambda_u^l - \kappa_u^l)(\phi^l - \phi^{l+1})\big)$$

RMI

Normalized mass flux

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Basic assumptions :

– M_u increases with the entrainment, detrainment is negligible where $\frac{\partial \omega_u^{\diamond\|}}{\partial p} > 0$;
– σ_u remains constant (at $\overline{\omega} \sim 0$) elsewhere , where there is detrainment.
– build $\mu \sim \mu^*$ from the bottom up, with $\mu = 1$ at lowest base
– assign a weight to the other (sub)-bases related to the integrated buoyancy of the associated segment

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- assign a weight to the other (sub)-bases related to the integrated buoyancy of the associated segment

$$\mu'^l = \begin{cases} \text{if } \delta_{\text{bas}}^l = 1: & \text{base weight} \\ \text{if } \delta_{\text{asc}}^l = 1: & \begin{cases} \mu^{*l+1}\exp\{\lambda_u^l(\phi^l - \phi^{l+1})\} & \text{if } \left(\frac{F}{K}\right)^l > \left(\frac{F}{K}\right)^{l+1} \\ \mu^{*l+1}\frac{\omega_u^{\diamond\|l}}{\omega_u^{\diamond\|l+1}} & \text{otherwise} \end{cases} \\ \text{if } \delta_{\text{asc}}^l = 0: & 0 \end{cases}$$

Steady-state $\omega_u^{\diamond\|}$

Auto-advection is important where buoyancy is small, and inversely.

$$\omega_u^{\diamond\|^2} - \frac{F}{K}(1 - \mathbf{b}\sigma_u)(1-\sigma_u)^2 - \frac{\beta}{K}\omega_u^{\diamond\|}(1-\sigma_u)^2 = 0$$

$$-\omega_u^{\|l} \approx \begin{cases} \max\left(\sqrt{F/K},\ \frac{-\omega_u^{\|l+1}}{1+K(p^{l+1}-p^l)}\right) & \text{if } \frac{F}{K} \geq 0 \\ \max\left(0,\ -\sqrt{-F/K} + \frac{-\omega_u^{\|l+1}}{1+K(p^{l+1}-p^l)}\right) & \text{if } \frac{F}{K} < 0 \end{cases}$$

In addition : prevent σ_u to decrease just above the base.

Base level has small buoyancy – base entrainment must be accounted for.

Local steady-state mesh fraction

$$g(\sigma, b, c) = \sigma(1-\sigma)\sqrt{1-b\sigma} + c\sigma = \frac{-M_B\mu}{\sqrt{F/K}}, \qquad c = \frac{-\overline{\omega}}{\sqrt{F/K}}$$

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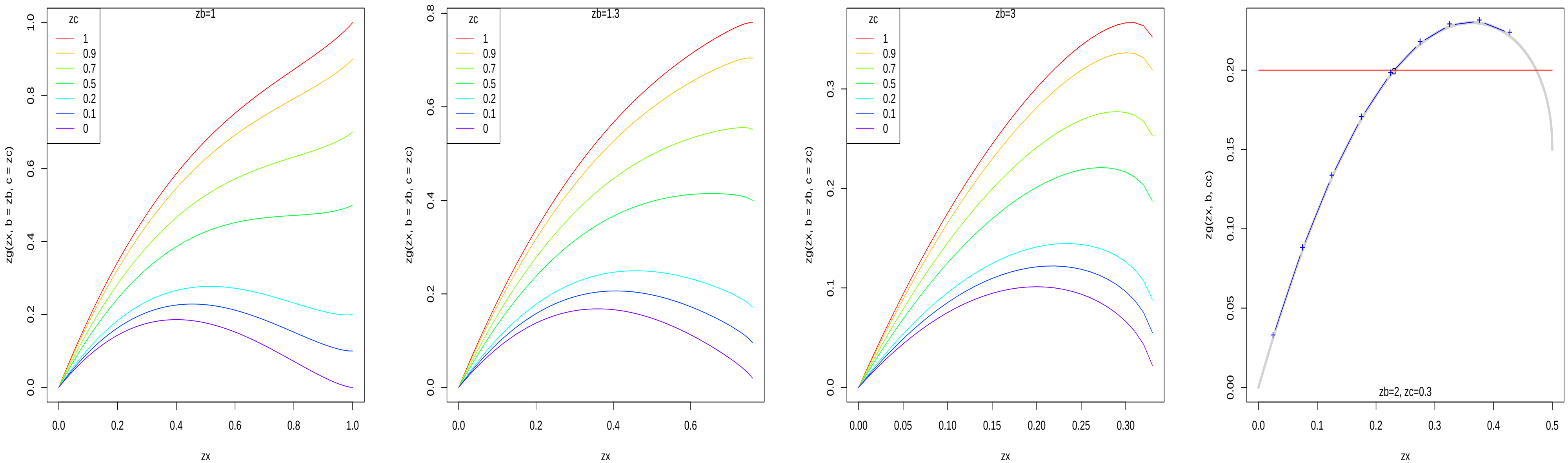

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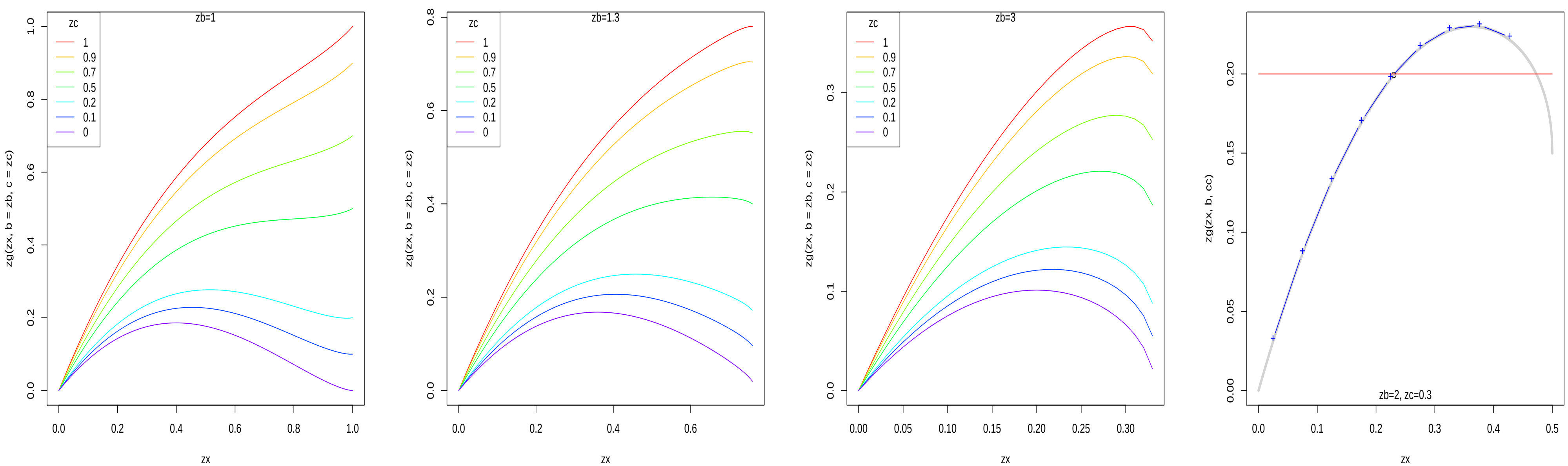

+ forbid σ_u bigger than the one of the maximum for $c = 0$.

Mesh fraction evolution : prognostic closure

Mesh fraction profile given by $\nu = \sigma_u^{\|}/\sigma_B^{\|}$.

$$\sigma_B^{\|} = \frac{\sum \sigma_u^{\|k} \triangle p^k \delta_{\rm sca}^k}{\sum \triangle p^k \delta_{\rm sca}^k}$$

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$$\sigma_B^{\|} = \frac{\sum \sigma_u^{\|k} \triangle p^k \delta_{\text{sca}}^k}{\sum \triangle p^k \delta_{\text{sca}}^k}$$

A prognostic evolution is still possible - limiting it to steady-state $\sigma_u^{\|}$.

$$\frac{\partial \sigma_B}{\partial t} \int_{p_t}^{p_b} \nu (h_u - h_e) \frac{dp}{g} = \sigma_B \int_{p_t}^{p_b} L\nu (\omega_u^{\diamond\prime} + \overline{\omega}) \frac{\delta q_{ca}}{g} + \int_{p_t}^{p_b} L \cdot \mathsf{CVGQ} \frac{dp}{g}$$

$$\omega_u^{\diamond\prime} = \beta \omega_u^{\diamond\|}, \qquad \beta = \frac{\sum \omega_u^{\diamond k} \delta p^k \delta_{\text{act}}^k}{\sum \omega_u^{\diamond\| k} \delta p^k \delta_{\text{act}}^k}$$

$$\sigma_B^- = \langle \sigma_u^{l-} > \epsilon_\sigma \rangle \qquad \sigma_u^{l+} = \min(\sigma_u^{\|l}, \nu^l \sigma_B^+)$$

RMI

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$$\omega_u^{\diamond\prime} = \beta \omega_u^{\diamond\|}, \qquad \beta = \frac{\sum \omega_u^{\diamond k} \delta p^k \delta_{\mathrm{act}}^k}{\sum \omega_u^{\diamond\|k} \delta p^k \delta_{\mathrm{act}}^k}$$

$$\sigma_B^- = \langle \sigma_u^{l-} > \epsilon_\sigma \rangle \qquad \sigma_u^{l+} = \min(\sigma_u^{\|l}, \nu^l \sigma_B^+)$$

σ_u memory not limited to active levels (for retrieving the norm σ_B).

δ_{ac9} obtained from $\omega_u^{\diamond -} < -\epsilon$.

RMI

Transport fluxes

$$J_\psi^{\mathrm{conv}\,l} = \frac{1}{g} \underbrace{\sigma_u \omega_u^*}_{-M_t} (\psi_u - \overline{\psi}), \qquad \frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial p} M_t (\psi - \psi_u) = -g \frac{\partial J_\psi^{\mathrm{conv}}}{\partial p}$$

RMI

Transport fluxes

$$J_\psi^{\mathrm{conv}\,l} = \frac{1}{g}\underbrace{\sigma_u\omega_u^*}_{-M_t}(\psi_u - \overline{\psi}), \qquad \frac{\partial\psi}{\partial t} = -\frac{\partial}{\partial p}M_t(\psi - \psi_u) = -g\frac{\partial J_\psi^{\mathrm{conv}}}{\partial p}$$

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The travel time $(\xi^{\overline{l-1}} - \xi^{\bar{l}})\triangle t$ induces a deposition corresponding to cloud (ψ_u) creation.

Transport flux is

$$\xi M_t = -\xi^{\bar{l}}\Big[\frac{\sigma_u}{1-\sigma_u}\frac{\omega_u^{\diamond+} + \omega_u^{\diamond-}}{2}\triangle t\Big]^{\bar{l}} = \xi^{\bar{l}}c^{\bar{l}} = \xi^{\bar{l}}\mathsf{ZFORM}^{\bar{l}} \geq 0$$

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$$J_\psi^{\mathrm{conv}\,\bar{l}} = \frac{(\xi c)^{\bar{l}}}{\triangle p^l + (\xi c)^{\bar{l}}}\left\{J_\psi^{\mathrm{conv}\,\overline{l-1}} + \frac{\triangle p^l}{g\triangle t}\left(\frac{\psi^{l+1} + \psi^l}{2} - \frac{\psi_u^{l+1} + \psi_u^l}{2}\right)\right\}$$

RMI

Condensation fluxes

Convective condensation associated to

$$M_c = \left[\sigma_u \frac{\omega_u^{\diamond +} + \omega_u^{\diamond -}}{2} \triangle t\right]^{\bar{l}} = d^{\bar{l}} = \mathsf{ZFORA}^{\bar{l}} \geq 0$$

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Variation of ξ cannot produce condensation.

$$\frac{\delta q_c}{\delta t} = (-\sigma_u \omega_u^{\diamond})\xi \frac{\triangle q_{ca}}{\triangle p} = -\frac{\delta q_v}{\delta t}$$

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Vertical transport ignored effect of condensation on $\overline{\psi}$:
include this transport in the condensation flux.

$$(\psi_*^l - \psi^l) = \frac{1}{\triangle p^l + (\xi c)^{\overline{l-1}}} \Big\{ (\xi c)^{\overline{l-1}} \big(\psi *^{l-1} - \psi^{l-1}\big) + \frac{(\xi d)^{\overline{l-1}} (\psi_u^l - \psi_u^{l-1}) + (\xi d)^{\bar{l}} (\psi_u^{l+1} - \psi_u^l)}{2} \Big\}$$

Detrainment area fraction

Local condensate budget within the subgrid updraught.

$$
\left.\begin{array}{ll}
\text{* condensate generation} & \propto \xi^l d^l \triangle q_{ca} \\[1em]
\text{* } \textit{inside} \text{ transport} & \propto \xi^l \sigma_u^l (\omega_u^\diamond + \overline{\omega_u})^l \\[1em]
\text{* entrainment} & \propto \xi^l c^l \lambda_u^l \overline{q_c}^l \\[1em]
\text{* local storage} & \propto \left(\delta_{\mathrm{act}} \sigma_u^{l+} - \delta_{\mathrm{ac9}} \sigma_u^{l-}\right) q_{cu}^l \\[1em]
\text{* detrained condensate} & \propto D\, \xi^l \triangle t\, q_{cu} = \delta \sigma_D\, q_{cD}
\end{array}\right\} \text{ where } \xi^l \to 0 \text{ at the rising top}
$$

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$$

Assuming $q_{cD} \approx q_{cu}$ not satisfactory

— pure mass budget must be further assessed.